



International Mathematics Competition 2008 (IMC 2008)

Individual Contest

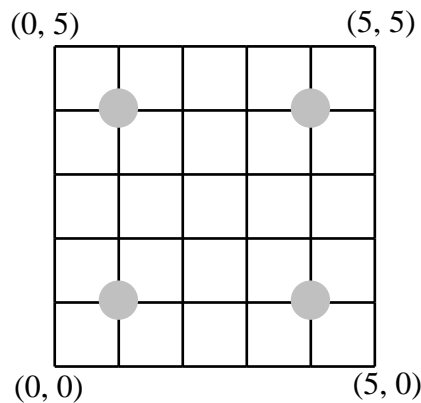
Time limit: 120 minutes 2008/10/28

Team: _____ Name: _____ Score: _____

Section A.

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

1. Starting from the southwest corner (0,0) of a 5×5 net, an ant crawls along the lines towards the northeast corner (5,5). It can only go east or north, but cannot get pass the four broken intersections at (1,1), (1,4), (4,1) and (4,4). What is the total number of different paths?



Answer : _____

2. The positive integer $a - 2$ is a divisor of $3a^2 - 2a + 10$. What is the sum of all possible values of a ?

Answer : _____

3. Let a, b and c be real numbers such that $a + b + c = 11$ and $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{13}{17}$. What

is the value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$?

Answer : _____

4. Let x be any real number. What is the maximum real value of $\sqrt{2008-x} + \sqrt{x-2000}$?

Answer : _____

5. How many ten-digit numbers are there in which every digit is 2 or 3, and no two 3s are adjacent?

Answer : _____

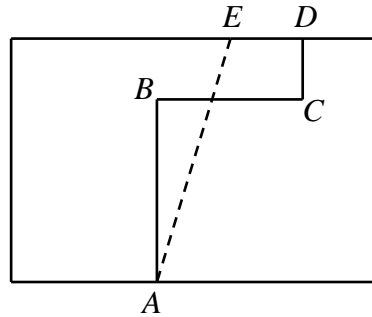
6. On a circle, there are n ($n > 3$) numbers with a total sum of 94, such that each number is equal to the absolute value of the difference between the two numbers which follow it in clockwise order. What is the possible value of n ?

Answer : _____

7. If the thousands digit of a four-digit perfect square is decreased by 3 and its units digit is increased by 3, the result is another four-digit perfect square. What is the original number?

Answer : _____

8. Each segment of the broken line $A-B-C-D$ is parallel to an edge of the rectangle, and it bisects the area of the rectangle. E is a point on the perimeter of the rectangle such that AE also bisects the area of the rectangle. If $AB=30$, $BC=24$ and $CD=10$, what is the length of DE ?



Answer : _____

9. Let $f(x) = ax^2 - c$, where a and c are real numbers satisfying $-4 \leq f(1) \leq -1$ and $-1 \leq f(2) \leq 2$. What is the maximum value of $f(8)$?

Answer : _____

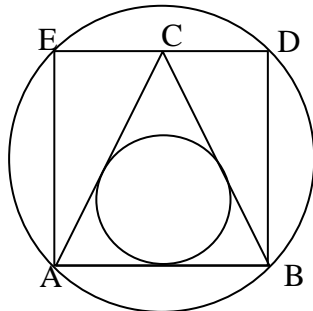
10. Two vertical mirrors facing each other form a 30° angle. A horizontal light beam from source S parallel to the mirror WV strikes the mirror UV at A , reflects to strike the mirror WV at B , and reflects to strike the mirror UV at C . After that, it goes back to S . If $SA=AV=1$, what is the total distance covered by the light beam?

Answer : _____

11. Let n be a positive integer such that $n^2 - n + 11$ is the product of four prime numbers, some of which may be the same. What is the minimum value of n ?

Answer : _____

12. ABC is an equilateral triangle, and $ABDE$ is a rectangle with DE passing through C . If the circle touching all three sides of $\triangle ABC$ has radius 1, what is the diameter of the circle passing through A , B , D and E ?



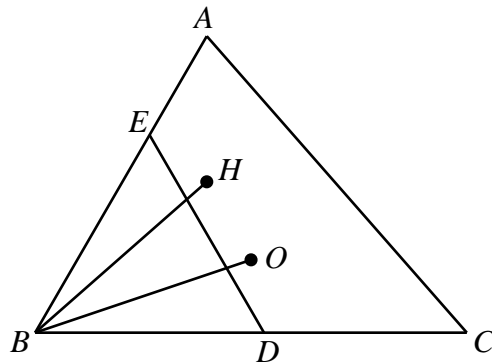
Answer : _____

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. In the expression $\left[\sqrt{2008 + \sqrt{2008 + \sqrt{2008 + \dots + \sqrt{2008}}} \right]$, the number 2008 appears 2008 times, and $[x]$ stands for the greatest integer not exceeding x . What is the value of this expression?

2. In the triangle ABC , $\angle ABC=60^\circ$. O is its circumcentre and H is its orthocenter. D is a point on BC such that $BD=BH$. E is a point on AB such that $BE=BO$. If $BO=1$, what is the area of the triangle BDE ? (The orthocenter is the intersection of the lines from each vertex of the triangle making a perpendicular with its opposite sides. The circumcenter is the center of the circle passing through each vertex of the triangle.)



3. Let t be a positive integer such that $2^t = a^b \pm 1$ for some integers a and b , each greater than 1. What are all the possible values of t ?