May 06, 2010

Dear Student,

I have the pleasure of informing you that you have been selected to the Sri Lankan Mathematics Challenge Competition 2010 (SLMCC 2010) based on your performance at the Sri Lankan Mathematics Competition 2010 held on April 24, 2010. SLMCC 2010 will be held at the University of Colombo from 9.00 a.m. to 1.30 p.m. on May 15, 2010. This competition will have 5 essay type problems. The best 12 at this competition will be awarded medals; Gold, Silver and Bronze in the ratio of 1 : 2 : 3. SLMCC 2010 paper and IMO Team Selection Competition 2010 are attached.

As you know the Sri Lankan team of six students that will represent Sri Lanka at the 51th International Mathematical Olympiad scheduled to be held in Astana, Kazakhstan during July 02 – 14, 2010 will be selected from this competition.

Books titled *Easy Olympiad Maths* containing solutions to the SLMCC papers 2004 to 2008 and other problems, a chapter on methods of proof, and a chapter on theorems and results in Olympiad mathematics; and *Introduction to Olympiad Maths* are available at S. Godage, No 661, P.D.S. Kularathna Mawatha, Maradana Road, Colombo 10 (Telephone: 112 686 925, Fax: 112 674 187, Email: info@godage.com, Web: www.godage.com). It is expected that you will study these books and be prepared for the competition. Please note that the competition paper has **one easy** problem, **two medium** problems and **two hard** problems and the time duration is **4.30 hours**.

There will be a seminar on problem solving on May 22 & 23, 2009 at the University of Colombo from 9.00 a.m. to 4.30 p.m. All of you are invited to the seminar but only the best 12 at SLMCC 2010 will be invited to the IMO Team Selection Competition 2010, scheduled to be held at the University of Colombo from 9.00 a.m. to 1.30 p.m. on May 29, 2010.

On the competition day May 15, 2010 please come to the Department of Mathematics, University of Colombo by 8.30 a.m. Please bring this letter and, your National Identity Card or Postal Identity Card or Passport. Please text 072 3678215 and confirm your participation. Good luck!

Yours truly,

Chanakya J. Wijeratne
CEO / Sri Lanka Olympiad Mathematics Foundation &
Former Head & Senior Lecturer/ Department of Mathematics, University of Colombo

Sri Lankan Mathematics Challenge Competition 2009

May 02, 2009

Time allowed: Four and a half hours
Instructions:

- *Full written solutions-not just answers are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*
- *One complete solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of problems than to try all five problems.*
- *Each problem carries 100 marks.*
- *Calculators and protractors are forbidden.*
- *Start each problem on a fresh booklet. Write on one side of paper only. On each booklet, write the number of the problem in the top left hand corner, and your name in the top right hand corner.*
- *Return all the booklets after the exam is over. Leave your rough work. You can take the exam home.*

Problem 1 Positive integers between 1 and 2009, inclusive of 1 and 2009, are ordered in a sequence. With this sequence we perform the following operation: if the first number in the sequence is *k*, we reverse order of the first *k* numbers in the sequence. Can 1 appear as the first term of the sequence after finitely many such operations for any original ordering of the sequence?

Problem 2 Let $ABCD$ be a convex quadrilateral with $\angle CBD = 2\angle ADB$, $\angle ABD = 2\angle CDB$, and $AB = CB$. Prove that $AD = CD$.

Problem 3 Prove that $\binom{n}{p} - \left\lfloor \frac{n}{p} \right\rfloor$ is divisible by p for every prime number p and every positive integer $n \geq p$.

Problem 4 Let $ABCD$ be a parallelogram with $\angle DAB = 60°$. Let O be the circumcenter of triangle ABD. Line AO intersects the external angle bisector of angle BCD at K. Find the value of $\frac{AO}{OK}$.

Problem 5 An exam paper consists of 5 multiple choice questions, each with 4 different choices. 2000 students take the exam, and each student chooses exactly one answer per question. Find the smallest value of *n* for which it is possible for the students' answer sheets to have the following property: among any *n* of the students' answer sheets, there exists 4 of them among which any two have at most 3 common answers.

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IMO Team Selection Competition 2009

May 23, 2009

Time allowed: Four and a half hours
Instructions:

- *Full written solutions-not just answers are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*
- *One complete solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of problems than to try all five problems.*
- *Each problem carries 100 marks.*
- *Calculators and protractors are forbidden.*
- *Start each problem on a fresh booklet. Write on one side of paper only. On each booklet, write the number of the problem in the top left hand corner, and your name in the top right hand corner.*
- *Return all the booklets after the exam is over. Leave your rough work. You can take the exam home.*

Problem 1 Find the greatest positive integer n such that the system of equations $(x+1)^2+y_1^2=(x+2)^2+y_2^2=...=(x+n)^2+y_n^2$ has an integral solution $(x, y_1, y_2, ..., y_n)$.

Problem 2 Let I_A be the centre of the escribed circle tangent to the side BC of triangle ABC. Denote by D the midpoint of the side AC and by E the intersection of the segments BC and I_AD. Prove that if $\angle BAC = 2\angle ACB$ then $|AB| = |BE|$.

Problem 3 An $n \times n$ chess board $(n \geq 2)$ is numbered by the numbers 1, 2, …, n^2 such that every number occurs. Prove that there exists two neighboring (with common edge) squares such that their numbers differ by at least n.

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