

Sri Lankan Mathematics Challenge Competition 2010

May 15, 2010

Time allowed: Four and a half hours

Instructions:

- *Full written solutions are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*
- *One complete solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of problems than to try all five problems.*
- *Each problem carries 100 marks.*
- *Calculators and protractors are forbidden.*
- *Start each problem on a fresh booklet. Write on one side of paper only. On each booklet, write the number of the problem in the top left hand corner, and your name in the top right hand corner.*
- *Return all the booklets after the exam is over. Leave your rough work. You can take the exam home.*

Problem 1 Numbers 1 to n^2 are written in some order in the unit squares of an $n \times n$ square such that in any rectangle consisting of some of those unit squares, the sum of the numbers in the two opposite corner squares equals the sum of the numbers in the other two opposite corners. Find all possible values for the sum of all numbers on a diagonal of the $n \times n$ square. Justify your answer.

Problem 2 At most how many distinct factors of 2009^{2010} can be selected such that none of the selected factors divides another selected distinct factor? Justify your answer.

Problem 3 Let A be the set of first 16 positive integers. Find the smallest positive integer k having the following property: In each subset of A with k elements there are two distinct numbers a, b such that $a^2 + b^2$ is prime. Justify your answer.

Problem 4 Consider a convex pentagon with the following properties: All of its sides are equal to 1 in length and some two of its diagonals intersect perpendicularly. Find the maximum possible area of such a pentagon. Justify your answer.

(Note: A pentagon is said to be convex if all of its interior angles are less than 180°)

Problem 5 Let P, Q be points on the sides AB and AC , respectively, of a triangle ABC , which satisfy $BP + CQ = PQ$. Let R be the point of intersection, other than A , of the bisector of the angle $\angle BAC$ and the circumcircle of the triangle ABC . If $\angle BAC = \alpha$, express $\angle PRQ$ in terms of α . Justify your answer.

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