Thank you very much for your participation in the Sri Lankan Mathematics Competition SLMC 2014. Your score on this competition will be posted against your index number in *www.slmathsolympiad.org*. The best 40 students in the SLMC 2014 will be invited to participate in the Sri Lankan Mathematics Challenge Competition 2014. In this competition we have tried to showcase mathematics by posing puzzle type questions covering various areas of mathematics. Though the problems require very little knowledge, not more than a Year 10 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your mathematical interest beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! For that you have to work hard. Solve math problems for fun - there are many websites in the internet and also good books featuring excellent mathematical problems - challenge yourself!!!

As you know doing these problems in the exam hall under the pressure of time is difficult. This way may not bring the best in you. We hope that you will leisurely think about these problems after the competition - Engaging in the problems and reflecting on them will improve your mathematical talent. Some of these problems have deep mathematical ideas in them. History shows us that some mathematical ideas we have to learn in school evolved through long periods of time baffling the greatest mathematical minds in those times. For example negative numbers, *Leo Rogers* says at http://nrich.maths.org/5961:

Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician *Brahmagupta*, it is surprising that in 1758 the British mathematician *Francis Maseres* was claiming that negative numbers "... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple".

Read that article. And don't be hard on yourself! Mathematics is a beautiful subject. We hope that this competition will help you to see the beauty in mathematics.

For any comments/suggestions: cjw@maths.cmb.ac.lk

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Medium | ENGLISH |

SRI LANKAN MATHEMATICS COMPETITION 2014

April 05, 2014

This question paper has **30 multiple choice questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

Scoring System for the Sri Lankan Mathematics Competition

Questions 1 to 10 : 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20 : 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30 : 8 points for correct response, 3 points for no response, and 0 points for incorrect response.

Sri Lanka Olympiad Mathematics Foundation

1. The least common multiple of three distinct numbers is 2014. The largest of these three numbers cannot be,

   (A) 53 (B) 106 (C) 1007 (D) 2014 (E) 4028

2. In base 2, five times 1101_2 equals,

   (A) 111101_2 (B) 111001_2 (C) 111111_2 (D) 1000001_2 (E) 1000011_2

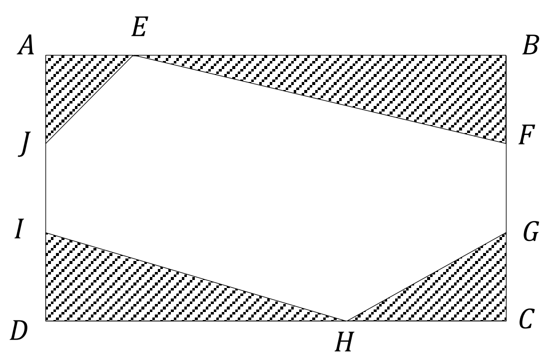
3. Time now is 10 pm. After $(24^{2014}+1)(24^{2015}+1)$ hours, the time will be,

   (A) 9 pm (B) 10 pm (C) 11 pm (D) 10 am (E) 11 am

4. If each of a_1, a_2 take values 2, 3 or 4 and $*$ is either addition or multiplication, how many different values can $a_1 * a_2$ take?

   (A) 8 (B) 9 (C) 11 (D) 12 (E) 13

5. In the rectangle $ABCD$, $AJ = JI = ID = BF = FG = GC$. What is the ratio, shaded area : unshaded area ?



   (A) 1 : 2
   (B) 2 : 1
   (C) 1 : 1
   (D) 1 : 3
   (E) 3 : 1

26. The value of $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}\ldots\ldots\ldots+\sqrt{1+\frac{1}{2013^2}+\frac{1}{2014^2}}$ is,

    (A) $2014+\frac{1}{2014}$ (B) $\frac{2013\times2015}{2014}$ (C) 2018 (D) $\sqrt{2013\times2014}$ (E) None given

27. Consider the sequence 1, 11, 111, 1111, ... , 111…111, … where the k^{th} term is k number of 1's and is denoted by a_k. Now consider following statements regarding the sequence,
    I. $a_{45^{2014}}$ is divisible by 41.
    II. $a_{2013\times2014}$ is not divisible by 91.
    III. There are terms which are divisible by both 41 and 91.
    Which of the above is/are true?
    (A) I and II only (B) I and III only (C) II and III only (D)All (E)None

28. The two parallel sides of a trapezium $ABCD$ are AB and CD. The midpoints of AB and CD are P and Q respectively. G is the intersection point of the two diagonals. Given that $CD = 2AB$, which of the following statements is/are true?
    I. The points P, G and Q are collinear.
    II. $8 \times$ Area of $PBG =$ Area of DGC
    III. $2AG = DG$

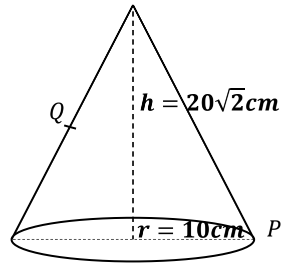
    (A) I and II only (B) I and III only (C) II and III only (D)All (E)None

29. For each positive integer n let $f(n)$ denote the smallest positive integer m such that nm is a perfect square. Consider the following statements.
    I. For all positive integers p, q; $\frac{f(p)f(q)}{f(pq)}$ is an integer.
    II. For each positive integer p, the progression $f(p), f(2p), \ldots, f(2^n p), \ldots$ is periodic.
    III. For each q positive integer there is a positive integer p such that $f(p) = q$.
    Which of the above statements is/are true?

    (A) I only (B) II only (C) I and II only (D) I and III only (E) All

30. Let C be a circle with unit radius ($r = 1$). Two *regular* polygons E and I, with 2014 sides are drawn such that, C touches the midpoints of each side of polygon E and each vertex of polygon I is on C. Assume E and I have perimeters of P_E and P_I respectively. Consider the following statements about E and I.
    I. $P_I < 2\pi < P_E$
    II. The vertices of E lie on the circle with centre that of C and radius $\frac{P_E}{P_I}$.
    III. The area of I is $\frac{P_I^2}{2P_E}$
    Which of the above are true?
    (A) I only (B) I and III only (C) II and III only (D) All (E) None

21. An ant crawls on the surface of a right circular cone shaped hat placed on a horizontal surface from the point P on the circular edge to the point Q which is the midpoint of the side (in front of P) on) on the triangle formed by vertical the cross section of the hat through P and the vertex of the cone. What minimum distance should the ant crawl?



(A) 15+10√2 (B) 15+5π (C) 15+10π (D) 15√3 (E) 20

22. There are six teams in a knock-out tournament. (A knockout tournament is a tournament where the loser of a game is eliminated from the tournament and there are no ties.) How many ways can the organizers arrange the tournament if only one match can be played at a time? *(For example, if the teams are A, B, C, D, E and F then one possible arrangement can be written as follows. A and B play the first game. Then the winner plays with C. The winner of that game plays with D and so on.)*

(A) 825 (B) 900 (C) 1024 (D) 1836 (E) 2700

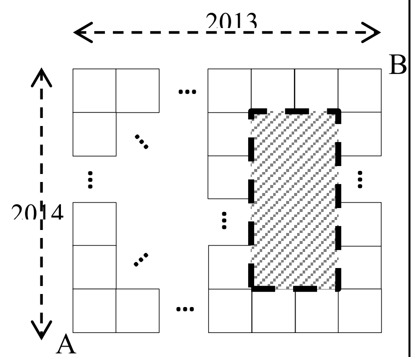
23. A bank manager uses a 4 digit password to access the bank system. Let's assume it is $abcd$. An employee found out that $a^2 + b^2 + c^2 + d^2 = 99$ and $a \times b \times c \times d = 36$. The system is built to automatically lock after 3 wrong attempts. Assume that the employee does not repeat a combination. What is the probability that the employee succeeds accessing the system using the given information?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{12}$ (D) $\frac{1}{16}$ (E) $\frac{1}{48}$

24. Take m to be the smallest positive number such that $5m$ is a perfect 5^{th} power, $6m$ is a perfect 6^{th} power and $7m$ is a perfect 7^{th} power. Which of the following is true about m?

(A) m has a prime divisor other than 2, 3, 5, or 7.
(B) $m > 210^{90}$
(C) $m < 5^{210}$
(D) m ends with exactly 35 zeros.
(E) None of the above is true.

25. From a 2014×2013 grid, a 2012×2 grid has been deleted as shown in the figure. How many paths are there **along the remaining grid** with the shortest distance from A to B?



(A) $\frac{4023!}{2009!2014!} + 1$ (B) $\frac{4027!}{2013!2014!} - 1007 \times 2013$ (C) $\frac{4024!}{2014!2010!} + 1$ (D) $\frac{4024!}{2014!2010!} + 2$ (E) Not given

6. How many four digit numbers are there such that all numbers obtained by rearranging its digits are divisible by 5?

(A) 1 (B) 8 (C) 16 (D) 648 (E) 1296

7. Which of the following is the sum of three consecutive integers?

(A) 2011^{2012} (B) 2012^{2013} (C) 2013^{2014} (D) 2014^{2015} (E) 2015^{2016}

8. In how many different ways can a 10×10 square grid be coloured using two colours in such a way that any two squares sharing exactly one vertex have different colours?

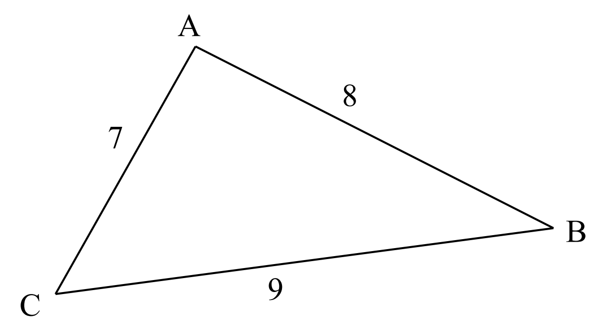
(A) 1 (B) 2 (C) 4 (D) 256 (E) 1024

9. When $p = \frac{2+\sqrt{2}}{3+\sqrt{2}}, q = \frac{\sqrt{7}}{\sqrt{8}}$ and $r = \frac{6}{7}$ are arranged in increasing order,

(A) $p < q < r$ (B) $r < p < q$ (C) $q < r < p$ (D) $q < p < r$ (E) $r < q < p$

10. Let ABC be a triangle with $AB = 8$, $BC = 9$, $CA = 7$. Let the lengths of altitudes of the triangle be AP, BQ and CR. Which of the following is true?

(A) $AP < BQ < CR$
(B) $AP < CR < BQ$
(C) $BQ < CR < AP$
(D) $CR < BQ < AP$
(E) None



11. Consider the following 3 statements:

Politicians are illogical (not logical). Friends of politicians are not disliked. Illogical persons are disliked.

What can you conclude?

I. Friends of politicians are illogical too.
II. Politicians are disliked.
III. Friends of politicians are logical.

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

12. *Isuru, Rusiru, Kasun, Manuja* and *Lajan* have registered themselves for Science, Maths, English, French, Dancing and Music exams (each of which is a three hour paper) as follows,

Isuru – Science, English, French
Rusiru – Maths, Dancing, English
Kasun – Science, Maths, Music
Manuja – Maths, English, Music
Lajan – Science, French, Dancing

If the exam is supposed to be held from 9 am to 12 noon and multiple exams can be held at the same time, what is the minimum number of days required to conduct all the six exams?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

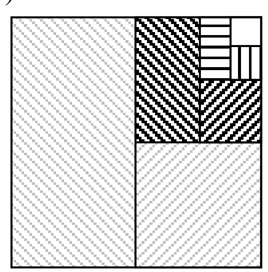
13. How many integers k are such that $x^{2014} + kx^2 + 4 = 0$ has an integer solution for x?

(A) 1 (B) 2 (C) 4 (D) None (E) Infinitely Many

14. A die (1×1×1 cube) is numbered in such a way that opposite faces get 1 & 6, 2 & 5, 3 & 4. Twenty seven of such dice are used to build a 3×3×3 cube. What is the maximum possible sum of face values of the cube? (Note that each face 9 values)

(A) 216 (B) 256 (C) 288 (D) 300 (E) 324

15. Look at the following figure and consider $S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \left(\frac{1}{2}\right)^n$ for any positive integer n. Which of the following statements are true?



I. $S_n < 1$ for each n.
II. $S_n = 1$ for some n.
III. $S_n > 2014$ for some n.

(A) I only (B) I and II only (C) II and III only (D) All (E) None

16. Let $x_1, x_2, \dots, x_n$ be n numbers where x_i is 0, 1 or 2 for each $i = 1, 2, \dots, n$ satisfying, $x_1 + x_2 + \cdots + x_n = 2013$ and $x_1^2 + x_2^2 + \cdots + x_n^2 = 2015$. What is the value of $x_1^3 + x_2^3 + \cdots + x_n^3$?

(A) 2014 (B) 2017 (C) 2018 (D) 2019 (E) More than one value

17. Suppose a rectangle has area 1 square meters. Then which of the following is/are true about the perimeter p of the rectangle?

I. p can be 3 meters.
II. p can be 4 meters.
III. For any positive integer n, p can take a value bigger than 2014^n meters.

(A) I only (B) II only (C) III only (D) II and III only (E) All

18. The quadrilateral $ABCD$ is such that $AC = BD$ and the points E, F, G and H are the midpoints of AB, BC, CD and DA respectively. Take I to be the point of intersection of AC and BD. Which of the following are true?

I. $\measuredangle HIG = 90°$
II. E, F, G and H lie on a circle.
III. $EH = EF = FG = GH$

(A) I only (B) II only (C) III only (D) II and III only (E) All

19. One of *Abdul, Mina, Kamala, Darian* or *Peter* broke a vase. Only one of these children always lies and others always tell the truth. This is what each of them had to tell about the person who broke the vase.

Abdul: Mina broke it.
Mina: Abdul lied.
Kamala: I did not break it.
Darian: I did not break it.
Peter: Abdul broke it.

Who broke the vase?

(A) Abdul (B) Mina (C) Kamala (D) Darian (E) Peter

20. What is the maximum number of perfect cubes within 1000 consecutive integers?

(A) 9 (B) 10 (C) 14 (D) 15 (E) 16