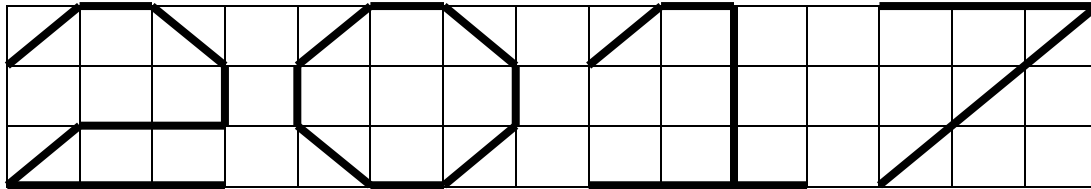


1. An artist has drawn 2017 in a square grid of side 1 cm squares using straight line segments as shown below. What is the total length of line segments in centimeters he has used?



- (A)  $20 + 10\sqrt{2}$  (B)  $21 + 10\sqrt{2}$  (C)  $20 + 11\sqrt{2}$  (D)  $21 + 11\sqrt{2}$   
 (E)  $21 + 12\sqrt{2}$
2. The smallest element in the set  $\left\{\frac{2017}{2018}, \frac{2018}{2019}, \frac{2019}{2020}, \frac{2020}{2021}, \frac{2021}{2022}\right\}$  is
- (A)  $\frac{2017}{2018}$  (B)  $\frac{2018}{2019}$  (C)  $\frac{2019}{2020}$  (D)  $\frac{2020}{2021}$  (E)  $\frac{2021}{2022}$
3.  $2017^2$  is not equal to
- (A)  $2017 \times 2018 - 2017$  (B)  $2016 \times 2017 + 2017$  (C)  $2016 \times 2018 + 1$   
 (D)  $2016 \times 2018 + 2$  (E)  $2015 \times 2019 + 4$
4. 2017 cubes of side 1 cm can be used to make a composite object by joining any two cubes by gluing two of their faces to coincide. What is the maximum possible surface area in square centimetres of such an object?
- (A)  $6 \times 2017 - 2 \times 2014$  (B)  $6 \times 2017 - 2016$  (C)  $6 \times 2017 - 2 \times 2016$   
 (D)  $6 \times 2017 - 2 \times 2017$  (E)  $6 \times 2017 - 2 \times 2018$
5. If  $A = \left\{\frac{2017}{2018}, \frac{2018}{2019}, \frac{2019}{2020}, \dots\right\}$  then which of following statements is/are true?
- I.  $A$  has infinitely many elements.  
 II.  $A$  does not have a largest element.  
 III. 1 is the largest element of  $A$ .
- (A) None (B) I only (C) I and II only (D) I and III only (E) All

27. Let  $S = \{1, 2, \dots, 3000\}$ . What is the least possible value of  $k$  such that any subset  $A$  of  $S$  with 101 elements contains two distinct numbers  $a$  and  $b$  such that  $|a - b| \leq k$

- (A) 29 (B) 39 (C) 49 (D) 59 (E) 69

28. In the *Land of Liars* everyone lies 6 days a week and tells the truth on the remaining day, and this day of the week is the same for each inhabitant. How many days of a week an inhabitant of the *Land of Liars* can make the following statement?

*If I did not tell the truth yesterday I will certainly tell the truth tomorrow.*

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

29. Which of the following statements is /are true for the sequence 1, 11, 111, 1111, ...?

- I. Every fourth term is divisible by 1111.  
 II. At least one term except the first term is a perfect fourth power.  
 III. Every  $3^n$  th term is divisible by  $3^n$ .

- (A) None (B) I only (C) I and II only (D) I and III only (E) All

30. In a  $100 \times 100$  grid a corner cell is colored red and all the other cells are colored blue. Consider the following recoloring operation: In any row or column colors of the cells are interchanged – red cells are colored blue and blue cells are colored red. Which of the following can be done through the recoloring operation?

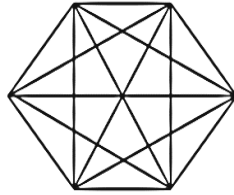
- I. All the corner cells red.  
 II. All the corner cells blue.  
 III. One of the corner cells blue and the other corner cells red.

- (A) I only (B) II only (C) III only (D) I and II only (E) All

23. What is the largest positive integer  $n$  such that there is a prime number  $p$  such that  $p, p + 2, p + 2^2, \dots, p + 2^n$  are all prime?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24. How many triangles are there in the following figure obtained by connecting all the vertices of a regular hexagon with straight line segments?



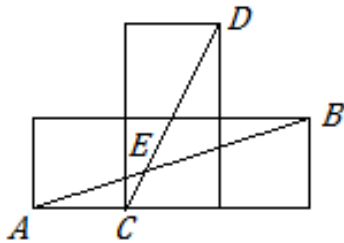
- (A) 80 (B) 90 (C) 100 (D) 110 (E) 120

25. In a  $100 \times 100$  grid a positive integer is written in each cell such that each number is the average of its neighbours; above, below, right and left. A number could have 2, 3 or 4 neighbours. Which of the following statements is/are true?

- I. All the numbers must be distinct.
- II. All the numbers must be same.
- III. Sum of all the numbers is divisible by 8.

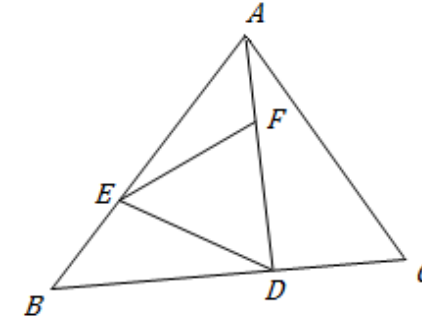
- (A) I only (B) II only (C) III only (D) I and III only (E) II and III only

26. In the following diagram each square has the side length  $1\text{cm}$ . If  $E$  is the intersection point of line segments  $AB$  and  $CD$  what is the measure of  $\angle DEB$  in degrees?



- (A)  $30^\circ$  (B)  $45^\circ$  (C)  $60^\circ$  (D)  $75^\circ$   
 (E) Cannot be determined from the given information

6. In the triangle  $ABC$ ,  $D, E$  and  $F$  are points on the line segments  $BC, AB$  and  $AD$  respectively, such that  $BD:DC = AE:EB = DF:FA = 2:1$ . What is  $\frac{\text{Area of } ABC}{\text{Area of } DEF}$ ?



- (A) 3 (B)  $1/3$  (C)  $8/27$  (D)  $27/8$  (E)  $16/27$

7. If the mode of the positive integers  $5, 6, 7, x, x$  equals their mean what is the mode?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

8.  $a_1, a_2, a_3, \dots, a_{100}$  are non-negative integers such that

$$(a_1 + a_3 + \dots + a_{99})(a_2 + a_4 + \dots + a_{100}) = 2017.$$

What is the sum  $a_1 + a_2 + a_3 + \dots + a_{100}$ ?

- (A) 1008 (B) 2016 (C) 2017 (D) 2018 (E) 4034

9. What is the range of the values  $a, 6, 6.5, 7, 12, 2a$  given in the increasing order?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

10. Among the attendees at a Sinhala and Tamil New Year lunch the ratio of Sinhala men to Sinhala women is  $2 : 3$  and the ratio of Tamil men to Tamil women is  $3 : 2$ . What is the ratio of men to women among the Sinhala and Tamil adult attendees if there is an equal number of Sinhala men and Tamil men?

- (A)  $6:9$  (B)  $25:12$  (C)  $12:25$  (D)  $13:12$  (E)  $12:13$

11. Each cell of a  $10 \times 10$  grid can be colored in red or blue. If each cell is distinct how many different coloring are there?
- (A) 100      (B) 200      (C)  $2^{10}$       (D)  $2^{20}$       (E)  $2^{100}$
12. Let  $m$  and  $n$  be positive integers such that  $3m + 7n$  is divisible by 11. Which of the following is always divisible by 11?
- (A)  $m + n$       (B)  $m + n + 5$       (C)  $9m + 4n$       (D)  $4m - 9n$       (E)  $6m + 4n$
13. A game is played by tossing a fair coin several times. If the outcome is a head, the player earns 1 point. If the outcome is a tail, the player earns  $-1$  point. Which of the following statements is/are true?
- I. It is not possible to earn 5 points by tossing the coin 10 times.  
 II. There is only one way (one sequence of heads and tails) to earn 3 points by tossing the coin 5 times.  
 III. It is possible to earn 0 points only in an even number of tosses.
- (A) I only      (B) II only      (C) III only      (D) I and II only      (E) I and III only
14. Five positive integers are such that any three numbers add up to at least 60. Which of the following statements is/are true?
- I. Each integer is at least 20.  
 II. Product of three of the given integers is at least  $20^3$ .  
 III. Product of any three of the given integers is at least  $20^3$ .
- (A) I only      (B) II only      (C) I and II only      (D) II and III only  
 (E) All
15. The Martian alphabet has only 2 letters:  $X$  and  $Y$ . A word in the Martian language is an arbitrary sequence of letters. How many different words of at most 5 letters are there in the Martian language?
- (A) 30      (B) 32      (C) 42      (D) 52      (E) 62

16.  $ABC$  is an equilateral triangle of side 10 cm and  $P$  is a point in the space. What is the minimum value in centimeters of  $3PA + 4PB + 7PC$ ?
- (A) 50      (B) 60      (C) 70      (D) 80      (E) 90
17. In the Martian language (see question 15) how many different words are there consisting of  $X$  ten times and  $Y$  five times such that between any two  $Y$ 's there is at least one  $X$ ?
- (A) 221      (B) 442      (C) 462      (D) 542      (E) 562
18. For each positive integer  $n$  let  $f(n) = k$ , where  $k$  is the largest positive integer with  $2^k$  divides  $n(n+1)(n+2)$ . Which of the following statements is/are true?
- I. For each positive integer  $m$ , there is a positive integer  $n$  such that  $f(n) = m$ .  
 II. There is a positive integer  $n$  such that  $f(n) = f(n+1)$ .  
 III. There are infinitely many positive integers  $n$  such that  $f(n) = 1$ .
- (A) I only      (B) II only      (C) III only      (D) I and II only      (E) I and III only
19. Number 1 is written 100 times on a whiteboard. Consider the following operation: Cross out any two numbers and write 0 if they are equal and 1 if they are not equal. What is the last number that will be written on the board?
- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4
20. What is the greatest integer that divides  $p^4 - 1$  for every prime number  $p$  greater than 5?
- (A) 12      (B) 30      (C) 48      (D) 120      (E) 240
21. How many subsets  $\{a, b\}$  of  $\{1, 2, \dots, 10\}$  are there such that  $a < b - 4$ ?
- (A) 10      (B) 15      (C) 20      (D) 25      (E) 30
22. Let  $A = \{1, 2, 3, 4\}$  and  $f$  an assignment on  $A$  such that  $f(i) \in A$  for  $i = 1, 2, 3, 4$  and  $f(f(i)) = i$  for each  $i \in A$ . How many such assignments are there?
- (A) 9      (B) 10      (C) 12      (D) 13      (E) 19