Thank you very much for your participation in the Sri Lankan Mathematics Competition SLMC 2018. Your score on this competition will be posted against your index number in *www.slmathsolympiad.org*. The best 30 students in the SLMC 2018 will be invited to participate in the Sri Lankan Mathematics Challenge Competition 2018. In this competition we have tried to showcase mathematics by posing puzzle type problems covering various areas of mathematics. Though the problems require very little knowledge of mathematics, not more than a Year 10 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your interest in mathematics beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! You have to learn problem solving strategies. Solve math problems for fun. Doing mathematics outside the school curriculum box will greatly improve your school mathematics.

As you know doing these problems in the exam hall under the pressure of time is difficult. This way may not bring the best in you. We hope that you will leisurely do and think about these problems after the competition. Looking back at the problems you solved and reflecting on them will improve your mathematical thinking. Some of these problems have deep mathematical ideas in them. History shows us that some mathematical ideas we have to learn in school evolved through long periods of time baffling the greatest mathematical minds in those times. For example negative numbers. Leo Rogers says at http://nrich.maths.org/5961:

Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician Brahmagupta, it is surprising that in 1758 the British mathematician Francis Maseres was claiming that negative numbers "... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple".

Read that article. Mathematics is a beautiful subject. But to see the beauty you have to engage in good mathematics. We hope that this competition will help you to see the beauty in mathematics.

For any comments/suggestions: Dr. Chanakya Wijeratne (cjw@maths.cmb.ac.lk)

Index No _____

Medium **ENGLISH**

SRI LANKAN MATHEMATICS COMPETITION 2018

April 7, 2018
10:30 am – 12 noon

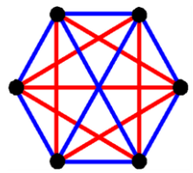
This question paper has **30 multiple choice questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

Scoring System for the Sri Lankan Mathematics Competition

Questions 1 to 10 : 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20 : 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30 : 8 points for correct response, 3 points for no response, and 0 points for incorrect response.



Sri Lanka Olympiad Mathematics Foundation

1. The following is a coded number coded using dancing men where each dancing man represents a unique digit (0, 1, 2, 3, 4, 5, 6, 7, 8 or 9) and different dancing men represent different digits:

Which of the following could be the correctly decoded number?

(A) 201877841637 (B) 201877841222 (C) 201877841333

(D) 201877841444 (A) 201877841555

2. In how many different ways can a nonempty plate be made from 2 identical *Kavuns* and 3 identical *Laddus*?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

3. What is the largest number that can be made by using each digit 2, 0, 1 and 8 of 2018 exactly once and the operations addition and multiplication any number of times?

(A) 10 (B) 16 (C) 18 (D) 24 (E) 25

4. How many different (non congruent) triangles are there with one side 4 cm and area 2018 cm^2?

(A) 1 (B) 2 (C) 3 (D) 4 (E) Infinitely many

5. In the table given below, each of the symbols @, #, & and $ denotes a number. The sum of the numbers denoted by each symbol in a column is given below the column and the sum of the numbers denoted by each symbol in a row is given to the right of the row except in the second row. What is the missing number?

@	@	#	#	14
&	$	@	&	
#	#	@	#	15
&	#	$	@	16
21	13	12	18	

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

27. Sarath, Ragavan and Mohamed played in an exhibition carrom event. In this event the loser of each game had to sit out the next game. In total Sarath played 9 games, Ragavan played 14 games and Mohamed played 15 games in this event. Which of the following is/are true?

I. Sarath and Ragavan played the 13^{th} game.
II. Ragavan and Mohamed played the 15^{th} game.
III. There were altogether 19 games.

(A) I only (B II only (C) III only (D) II and III only (E) All

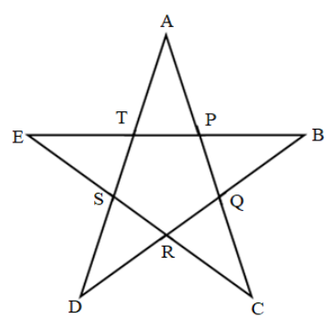
28. The increasing sequence 1, 3, 4, 9, 10, 12, 13, … consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. What is the 100^{th} term of this sequence?

(A) 981 (B) 2019 (C) 2018 (D) $2^{100} - 3^{10}$ (E) $3^{50} - 2^{50}$

29. a and b are positive integers such that a^b has exactly 5 positive divisors and b^a has exactly 7 positive divisors. How many positive divisors does ab have ?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

30. A pentagram is a regular pentagon with its sides extended to their point of intersection. In the pentagram *ABCDE* shown below *PQRST* is a regular pentagon.



If $AP: PQ = m : 1$, then what is the ratio ,the area of the pentagram : the area of the pentagon?

(A) $(5m + 2) : (m + 2)$ (B) $(m + 2) : (5m + 2)$ (C) $(6m + 2) : (m + 2)$

(D) $(m + 2) : (6m + 2)$ (E) $(6 + 2m) : (1 + 2m)$

21. Two players take turns in tossing a fair coin. The winner is the first one to toss a head and the game is played until there is a winner. What is the probability that the first player, the player who tosses first, wins the game?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

22. Each cell of a 10×10 square grid should be colored in red or blue such that each 2×2 square has an odd number of blue squares. If each cell is distinct then how many different colorings are there?

(A) 99 (B) 100 (C) 2^{10} (D) 2^{19} (E) 2^{99}

23. All integers from 1 to 2018 are written on a white board. Two players take turns in selecting two numbers and erasing them and writing their positive difference instead. If the last number written on the board is odd then the first player, the player who plays first, wins. Otherwise the second player wins. Which of the following is/are true?

I. The first player always wins.
II. The second player always wins.
III. The second player has a winning strategy.

(A) I only (B) II only (C) III only (D) II and III only (E) None

24. A set A consists of 2019 integers such that the sum of any 2018 elements of A is divisible by 2018. Which of the following statements is/are necessarily true?

I. Every element of A is divisible by 2018.
II. Every element of A has the same remainder when it is divided by 2018.
III. At least one element of A is divisible by 2018

(A) I only (B) II only (C) III only (D) All (E) None

25. How many 4 digit numbers are there such that any 4 digit number obtained by a rearrangement of its digits is divisible by 5? (Note that a number with 4 digits has its first digit of the four digits non zero.)

(A) 1 (B) 8 (C) 16 (D) 20 (E) 25

26. One of Kanchana, Damitha, Ranjan, Pooja and Roshan broke a vase. The following is what each of them had to say about the person who broke the vase:

Kanchana: If Ranjan did not break the vase then Pooja or Roshan broke it.
Damitha: Ranjan broke the vase.
Ranjan: Pooja broke the vase.
Pooja: Roshan broke the vase.
Roshan: Kanchana broke the vase.

If only one of these 5 statements is true, who broke the vase?

(A) Kanchana (B) Damitha (C) Ranjan (D) Pooja (E) Roshan

6. Which of the following statements is true for all real values (positive, negative and zero) of the number a?

(A) $7a > 3a$ (B) $7a^2 > 3a^2$ (C) $7(a+1) > 3(a+1)$ (D) $5 - 7a > 5 - 3a$
(E) $217 + a > 3 + a$

7. The hypotenuse of a right triangle is 2018 cm and a side is 1118 cm. What is the length of the other side in cm? (Note: $\sqrt{3136}$=56)

(A) 1675 (B) 1680 (C) 1685 (D) 1690 (E) 1695

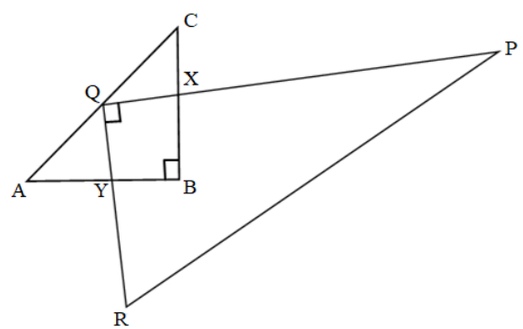
8. A regular triangle (equilateral triangle), a regular quadrilateral (square), a regular pentagon and a regular hexagon have interior angles measured in integer degrees. In total, how many types of regular polygons have interior angles measured in integer degrees?

(A) 20 (B) 22 (C) 23 (D) 26 (E) 30

9. In a primary school, the number of participants in a sports meet is 75. For the three events A, B and C of the sports meet, the participants are in the ratio 2 : 5 : 4 and each participant participated in at least one of these three events. Every student who participated in event A also participated in event B and no student participated in all three events. If the ratio of the number who participated in both events B and C to the number who participated in event A is 1:3, how many students participated in exactly one event?

(A) 50 (B) 51 (C) 52 (D) 53 (E) 54

10. In the figure given below (not drawn to scale), ABC is an isosceles right triangle with the two equal sides of length 2 cm each and PQR is a right triangle. Q is the midpoint of AC.



What is the area of the quadrilateral $XQYB$ in cm^2?

(A) 1 (B) 1.5 (C) 2 (D) 2.5 (E) 3

11. When Sumana's piggy bank broke, she discovered that the number of 5 rupee coins she had was three times the number of 10 rupee coins she had and that the value of the 2 rupee coins she had was three rupees more than the value of the 5 rupee coins she had. If Sumana had at least ten 2 rupee coins, what is the minimum amount that she must have had?

(A) 112 Rupees (B) 115 Rupees (C) 120 Rupees (D) 123 Rupees (E) 128 Rupees

12. Pasindu says that his six digits secret number is the smallest positive integer such that it is a multiple of 9 and if its first and last digits are crossed out then the remaining number has only 11 as a prime factor. What is Pasindu's secret number?

(A) 111114 (B) 122220 (C) 333333 (D) 113310 (E) 113319

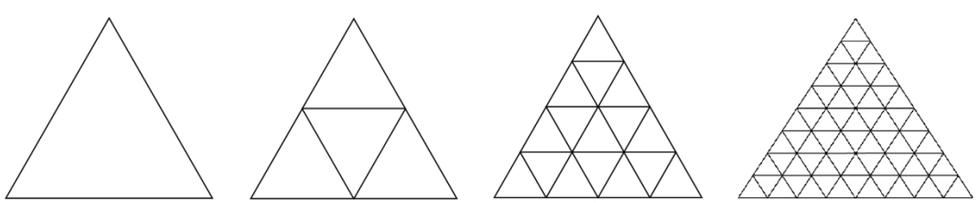
13. There are four keys and four locks. Each of the four keys fits exactly one of the four locks and different keys fit different locks. In the worst-case scenario what is the minimum total number of times a key has to be tested in a lock to match up all the keys and locks?

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

14. Champernowne constant in base 10 is 0.123456789101112… . This is the number obtained by concatenating the positive integers and interpreting the digits as decimal digits to the right of the decimal point. What is the 2018th digit to the right of the decimal point?

(A) 0 (B) 2 (C) 3 (D) 6 (E) 7

15.



The first four figures in a pattern are given above. The first figure in the pattern is an equilateral triangle. Each other figure in the pattern is obtained by joining the midpoints of the sides of the triangle/triangles in the previous figure with straight lines parallel to the sides. If the number of triangles of the smallest size in two successive figures add up to 20 480, which two figures in this pattern are they? (Note: $4^6 = 4096$)

(A) 4th and 5th (B) 5th and 6th (C) 6th and 7th (D) 7th and 8th (E) 8th and 9th

16. How many parallelograms are contained in the 4th figure in the pattern given in question 15 above? (Hint: symmetry)

(A) 620 (B) 625 (C) 630 (D) 635 (E) 640

17. If $\frac{1}{3}+\frac{1}{3}+\ldots+\frac{1}{3}=\frac{1}{5}+\frac{1}{5}+\ldots+\frac{1}{5}$ and the total number of fractions on both sides is between 20 and 26 then what exactly is the total number of fractions?

(A) 21 (B) 22 (C) 23 (D) 24 (E) 25

18. Let $a_1, a_2, a_3, \ldots, a_{2017}, a_{2018}$ be a sequence of real numbers such that $a_1 = 1$ and $a_{n+1} + a_n = a_{n+1}^2 - a_n^2$ for $n \geq 1$. What is the minimum possible value of a_{2018}?

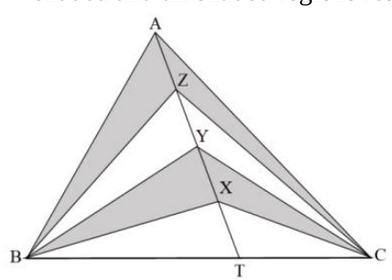
(A) -2018 (B) -2017 (C) 0 (D) 2017 (E) 2018

19. A rectangular chocolate bar of size $x \times y$ where x and y are positive integers consists of xy small squares. The bottom left square is bitter and the other squares are sweet. Two players take turns in selecting a square and eating all the squares above and to the right of the selected square in a rectangular block including the selected square. The player who has to eat the bitter square loses. Which of the following is/are true?

I. If $x \neq 1$ and $y \neq 1$ then the first player, the player who eats first, has a winning strategy.
II. If $x \neq 1$ and $y \neq 1$ then the second player has a winning strategy.
III. If ($x = 1$ and $y \neq 1$) or ($x \neq 1$ and $y = 1$) then the first player can win in a single move.

(A) I only (B) II only (C) I and III only (D) None (E) All

20. Consider the figure given below. T is a variable point on the side BC of triangle ABC. Points X, Y, Z are on AT such that $AZ = ZY = YX = XT$. Let S and U be the areas of the shaded and un-shaded regions respectively. Which of the following statements is/are true?



I. $S = U$ only when AT is perpendicular to BC.
II. $S = U$ only when T is the midpoint of BC.
III. $S = U$ for any position of T on the line segment BC.

(A) I only (B) II only (C) III only (D) None (E) All