



奧冠教育中心

OLYMPIAD CHAMPION EDUCATION CENTRE

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世界國際數學競賽總決賽 2018

WORLD INTERNATIONAL MATHEMATICAL OLYMPIAD FINAL 2018

小學六年級 Primary 6

時限：120 分鐘

Time allowed: 120 minutes

試題

Question Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 6 題，共 30 題，每題 5 分，總分 150 分，答錯不扣分。
There are 5 exam areas and 6 questions in each exam area. There are a total of 30 questions in this Question-Answer Book. Each carries 5 marks. Total score is 150 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
This Question-Answer Book will be collected at the end of the contest.

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

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 上。

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本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.

Otherwise, contestant may be **DISQUALIFIED**.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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填空题 (第 1 至 30 題) (每題 5 分, 答錯及空題不扣分)

Open-Ended Questions (1st ~30th) (5 points for correct answer, no penalty point for wrong answer)

Logical Thinking

邏輯思維

1. Mixing 3 bottles of solution with concentration 20%, 30% and 50% yields a bottle of 100L solution of concentration 36%. It is known that the volume of solutions of 30% concentration used is twice of that of the 20% one. How much solution of 30% concentration is used (in L)?
把濃度為 20%、30% 和 50% 的溶液混合在一起, 得到濃度為 36% 的溶液 100 升。已知濃度為 30% 的溶液用量是濃度為 20% 的溶液用量的 2 倍, 濃度為 30% 的溶液的用量是多少升?
2. It takes Amy 11 days and Fanny 9 days to complete $\frac{3}{8}$ of a task; it takes Amy 14 days and Fanny 16 days to complete $\frac{2}{3}$ of the same task. If Amy and Fanny work together, how many day(s) are needed to take for to complete the task?
一項工作, 小艾 做 11 天後、小芳 然後單獨做 9 天可以完成整份工程的 $\frac{3}{8}$; 而 小艾 做 14 天、小芳 然後單獨做 16 天可以完成整份工程的 $\frac{2}{3}$ 。那麼 小艾、小芳 兩人合作完成這份工程需要多少天?
3. Fruits are shared among students in class 6A, including apples, pears, oranges, strawberries and kiwis. There are 2000 fruits of each type available. If each student picks two fruits randomly, at least how many students are there so that we are sure that two students picked the exactly two same fruits?
6A 班同學分享水果, 有蘋果、梨、橙、草莓和奇異果 5 種, 每種各 2000 個。如果每位同學任意拿兩個, 那麼至少多少位同學拿過後才一定會出現兩人拿的水果是相同的。
4. There are three numbers A, B and C. The sum of A and B is 345. The sum of B and C is 456. The sum of A and C is 567. What is the average of these three numbers?
現有甲、乙、丙三數, 甲與乙之和是 345; 乙與丙之和是 456; 甲與丙之和是 567。請問這三數的平均數是多少?
5. If either 22 is added to or subtracted from a natural number, the results are also perfect square numbers. Find the minimum value of that natural number.
若對某個自然數, 加上 22 或減去 22 之後都會是完全平方數, 求該自然數的最小值。
6. Define an mathematical operation 「 \otimes 」 as $x \otimes y = \frac{1}{x(y-1)} + \frac{1}{x(y-A)}$. Given that $1 \otimes 2 = 2$.
Evaluate $2015 \otimes 2016$.
定義符號「 \otimes 」為一個計算程序符合 $x \otimes y = \frac{1}{x(y-1)} + \frac{1}{x(y-A)}$ 。已知 $1 \otimes 2 = 2$, 求 $2015 \otimes 2016$ 的

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請將答案寫在

答題紙

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值。

Arithmetic

算術

7. Find the value of $11+13+15+17+\dots+99-2015$.

求 $11+13+15+17+\dots+99-2015$ 的值。

8. Find the value of $\frac{1}{2 \times 3} + \frac{2}{3 \times 5} + \frac{3}{5 \times 8} + \frac{4}{8 \times 12} + \frac{5}{12 \times 17} + \frac{6}{17 \times 21} + \frac{7}{21 \times 29} + \frac{8}{29 \times 37} + \frac{9}{37 \times 46}$.

求 $\frac{1}{2 \times 3} + \frac{2}{3 \times 5} + \frac{3}{5 \times 8} + \frac{4}{8 \times 12} + \frac{5}{12 \times 17} + \frac{6}{17 \times 21} + \frac{7}{21 \times 29} + \frac{8}{29 \times 37} + \frac{9}{37 \times 46}$ 的值。

9. Find the value of $2015 + 0.\dot{2}01\dot{5} + 0.20\dot{1}\dot{5}$. (Express your answer in recurring decimals)

求 $2015 + 0.\dot{2}01\dot{5} + 0.20\dot{1}\dot{5}$ 的值。(答案以循環小數表示)

10. Find the value of $1001^2 - 997^2 + 993^2 - 989^2 + 985^2 - 981^2 + \dots + 809^2 - 805^2$.

求 $1001^2 - 997^2 + 993^2 - 989^2 + 985^2 - 981^2 + \dots + 809^2 - 805^2$ 的值。

11. Find the value of $\left(1 - \frac{1}{2}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 + \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 + \frac{1}{4}\right) \times \dots \times \left(1 - \frac{1}{939}\right) \times \left(1 + \frac{1}{939}\right)$.

求 $\left(1 - \frac{1}{2}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 + \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) \times \left(1 + \frac{1}{4}\right) \times \dots \times \left(1 - \frac{1}{939}\right) \times \left(1 + \frac{1}{939}\right)$ 的值。

12. Find the value of $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + 4 \times 5 \times 6 + \dots + 18 \times 19 \times 20$.

求 $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + 4 \times 5 \times 6 + \dots + 18 \times 19 \times 20$ 的值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Number Theory

數論

13. Find the sum of all positive factors of 2880.

求 2880 的所有正因數之和的值。

14. If $\overline{2015726ab}$ is a multiple of 99, find the value of \overline{ab} .

如果 $\overline{2015726ab}$ 是 99 的倍數，求 \overline{ab} 的值。

15. Find the value of $\frac{1}{2015 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1}}}}}}$. (Express your answer in fraction)

求 $\frac{1}{2015 + \frac{1}{4 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1}}}}}}$ 的值。(答案以分數表示)

16. Define $n! = n \times (n-1) \times \dots \times 3 \times 2 \times 1$. Find the last digit of $1! + 2! + 3! + \dots + 2015!$.

定義 $n! = n \times (n-1) \times \dots \times 3 \times 2 \times 1$ ，求 $1! + 2! + 3! + \dots + 2015!$ 的個位數。

17. Find the last digit of

$$1514^{13} + 1413^{12} + 1312^{11} + 1211^{10} + 1110^9 + 109^8 + 98^7 + 87^6 + 76^5 + 65^4 + 54^3 + 43^2 + 32^1.$$

求 $1514^{13} + 1413^{12} + 1312^{11} + 1211^{10} + 1110^9 + 109^8 + 98^7 + 87^6 + 76^5 + 65^4 + 54^3 + 43^2 + 32^1$ 的個位數。

18. Find the remainder when $\underbrace{222\dots222}_{2015}$ is divided by 41.

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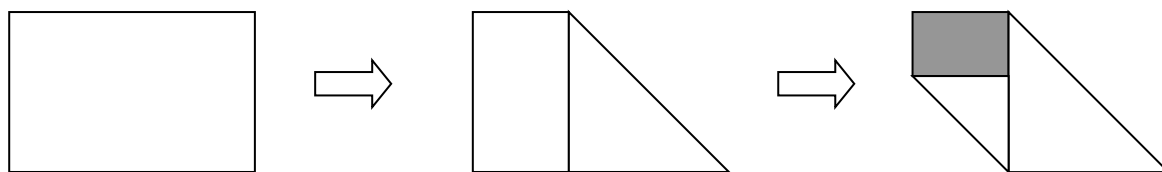
求當 $\underbrace{222\dots222}_{2015}$ 除以 41 的餘數。

Geometry

幾何

19. There is a rectangular paper with length 23cm and width 15cm. Then the paper is folded its upper right corner to the lower edge and is folded its lower left corner to meet the first triangle as shown in the figure below. Find the area of the shaded region.

有一張長 23 厘米，寬 15 厘米的長方形紙片，若以圖中的方法把它的右上角往下折疊，再把左下角往上折疊。求陰影部分的面積。



Question 19

第 19 題

20. The area of a rectangle is 750. The value of its length and width are both integers. What is the maximum possible value of its perimeter?

一個長方形面積是 750，而該長方形的長和闊的長度都是整數；那麼長方形的周界最大是多少？

21. The figure below shows a triangle $\triangle ABC$ which area is 1050cm^2 . If $AD:AB=1:5$, $BE:EC=1:4$, $CF:CA=1:5$, find the area of $\triangle DEF$.

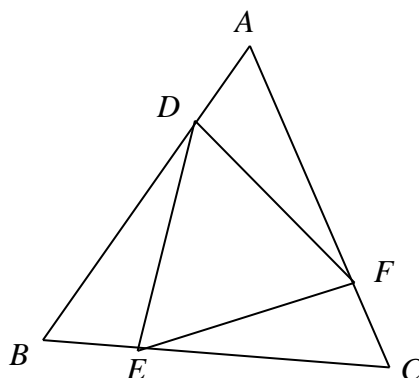
三角形 $\triangle ABC$ 的面積是 1050 平方厘米， $AD:AB=1:5$ ， $BE:EC=1:4$ ， $CF:CA=1:5$ ，求 $\triangle DEF$ 的面積。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Question 21

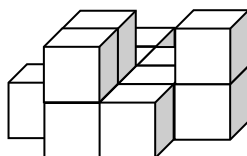
第 21 題

22. At most how many line(s) can be formed by using 25 points on a plane?

在平面上畫 25 點，最多可連成多少條線？

23. Refer to the figure below, the length of the edge of each smaller cube is 2. Find the surface area of the 3-D figure.

參考附圖，每個正方體的邊長都是 2，求立體的表面積。



Question 23

第 23 題

24. The figure shows a cylindrical bucket with some water inside. The base area is 128 cm^2 . The height of the water level is 9cm. Now a rectangular solid of length of 8cm, width of 8cm and height of 24cm is inserted vertically into the water. What is the height of water level at last? (Express the answer in cm)

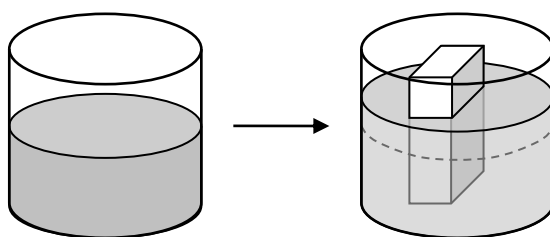
有一個裝了水的圓柱體水桶，桶底面積為 128 平方厘米，水高 9 厘米。現在將一個長和闊都是 8 厘米，高 24 厘米的長方體鐵塊放進桶裡，問最後水高多少厘米？

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Question 24

第 24 題

Combinatorics

組合數學

25. The average age for employees in department A, B, and C of a company are respectively 38, 24 and 42. The average age for employees in department A and B is 30. The average age for employees in department B and C is 34. What is the average age of all employees in the company?

A、B、C 三個部門人員平均年齡分別為 38 歲、24 歲、42 歲。A 和 B 兩部門人員平均年齡為 30 歲，B 和 C 兩部門人員平均年齡為 34 歲。問該公司部門人員平均年齡為多少歲？

26. There are in total of 18 animals. They are either spiders, dragonflies or cicadas. Spiders have 8 legs and no wings; dragonflies have 6 legs and 2 pairs of wings; Cicadas has 6 legs and 1 pair of wings. If there are in total 116 legs and 22 pairs of wings, how many cicada(s) is / are there?

有蜘蛛、蜻蜓和蟬三種昆蟲共 18 隻，它們共有腿 116 條，翅膀 22 對。問有多少隻蟬？（蜘蛛有 8 條腿，沒有翅膀；蜻蜓有 6 條腿，2 對翅膀；蟬有 6 條腿，1 對翅膀。）

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答題紙

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27. Amy, David and Mary together have to buy some office stationeries. Amy paid \$316 for 1 calculator, 3 staplers and 7 packs of paper; David paid \$362 for 1 calculator, 4 staplers and 10 packs of paper. How much does Mary need to pay for 1 calculator, 1 stapler and 1 pack of paper?

小美、小偉、瑪麗三人需購買辦公用品，小美購買 1 個計算機，3 個釘書機，7 包打印紙共需要 316 元；小偉購買 1 個計算機，4 個釘書機，10 包打印紙共需要 362 元；求瑪麗購買了 1 個計算機，1 個釘書機，1 包打印紙，共需付多少元？

28. If a and b are two different integers from 11 to 30 inclusively, find the minimum possible value of $\frac{4a+6b}{5a+7b}$.

如果 a 和 b 分別是 11 至 30 內含的兩個不同的整數，求 $\frac{4a+6b}{5a+7b}$ 的最小可能值。

29. A palindromic number is a whole number that reads the same from either directions. For example, 1991 and 23432 are two palindromic numbers. Find the value of the 19th 7-digit palindromic number in ascending order.

若某一整數的數位左右次序互換後數值不變，則稱該數為回文數，例如 1991 和 23432 均是回文數。求由小至大排列第 19 個 7 位回文數。

30. How many digits does the product of 22222222×22222 have?

22222222×22222 的積有多少個位？

~ 全卷完 ~

~ End of Paper ~

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