



奧冠教育中心

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世界國際數學競賽總決賽 2018

# WORLD INTERNATIONAL MATHEMATICAL OLYMPIAD FINAL 2018

中學二年級 Secondary 2

時限：120 分鐘

Time allowed: 120 minutes

試題

## Question Paper

考生須知：

### Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。  
Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 6 題，共 30 題，每題 5 分，總分 150 分，答錯不扣分。  
There are 5 exam areas and 6 questions in each exam area. There are a total of 30 questions in this Question-Answer Book. Each carries 5 marks. Total score is 150 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。  
All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。  
NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。  
All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。  
This Question-Answer Book will be collected at the end of the contest.

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 



 上。

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本試題不可取走。

**THIS Question-Answer Book CANNOT BE TAKEN AWAY.**

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

**DO NOT** turn over this Question-Answer Book without approval of the examiner.

Otherwise, contestant may be **DISQUALIFIED**.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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填空題 (第 1 至 30 題) (每題 5 分, 答錯及空題不扣分)

Open-Ended Questions (1<sup>st</sup> ~30<sup>th</sup>) (5 points for correct answer, no penalty point for wrong answer)

Logical Thinking

邏輯思維

1. It is known that the lengths of three sides of a right-angled triangle are integers and one of the sides other than the hypotenuse is 12. Find the number of triangle(s) satisfying the above mentioned conditions.

一個直角三角形的三條邊均為整數且其中一非斜邊的長度為 12，有多少個三角形的符合上述條件？

2. Given 
$$\begin{cases} a+b+c+d=4 \\ a+b+c-e=5 \\ a+b+c+d+e=7 \end{cases}, \text{ find the value of } \frac{a+b+c}{d+e}.$$

已知 
$$\begin{cases} a+b+c+d=4 \\ a+b+c-e=5 \\ a+b+c+d+e=7 \end{cases}, \text{ 求 } \frac{a+b+c}{d+e} \text{ 的值。}$$

3. Find the factor of 104060405 closest to 10000. (Hint: factorize  $x^4 + 4$ .)

求 104060405 最接近 10000 的因數。(提示：因式分解  $x^4 + 4$ 。)

4. It is known that  $(x^2 + 2x + a)(x^2 + bx + 1) \equiv (x^3 + c)(x + 1)$ , find the value of  $a + b + c$ .

已知  $(x^2 + 2x + a)(x^2 + bx + 1) \equiv (x^3 + c)(x + 1)$ ，求  $a + b + c$  的值。

5. Fruits are shared among students in class 8A, including apples, pears, oranges, strawberries, kiwis and watermelons. There are 4000 fruits of each type available. If each student picks two fruits randomly, at least how many students are there so that we are sure that two students picked the exactly two same fruits?

8A 班同學分享水果，有蘋果、梨、橙、草莓、奇異果及西瓜 6 種，每種各 4000 個。如果每位同學任意拿兩個，那麼至少多少位同學拿過後才一定會出現兩人拿的水果是相同的。

6. When a 3-digit number is multiplied by 15, the product has a remainder of 1 when it is divided by 7. Find the smallest possible value of this 3-digit number.

某三位數乘 15 後除以 7 餘 1，求該數的最小可能值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Algebra

代數

7. Given  $\begin{cases} \frac{a+1}{2} + \frac{b-7}{3} = 7 \\ \frac{a+1}{2} - \frac{b-7}{3} = 4 \end{cases}$ , find the value of  $a+2b$ .

若  $\begin{cases} \frac{a+1}{2} + \frac{b-7}{3} = 7 \\ \frac{a+1}{2} - \frac{b-7}{3} = 4 \end{cases}$ ，求  $a+2b$  的值。

8. If  $x^2 + 2x + 3 = 0$ , find the value of  $x^4 - 4x + 17$ .

若  $x^2 + 2x + 3 = 0$ ，求  $x^4 - 4x + 17$  的值。

9. Find the value of  $1^2 + 4^2 + 7^2 + \cdots + 49^2$ .

求  $1^2 + 4^2 + 7^2 + \cdots + 49^2$  的值。

10. Find the value of  $\frac{4 \times 8 + 8 \times 12 + 12 \times 16 + \cdots + 36 \times 40 + 40 \times 44}{2 \times 4 + 4 \times 6 + 6 \times 8 + \cdots + 18 \times 20 + 20 \times 22}$ .

求  $\frac{4 \times 8 + 8 \times 12 + 12 \times 16 + \cdots + 36 \times 40 + 40 \times 44}{2 \times 4 + 4 \times 6 + 6 \times 8 + \cdots + 18 \times 20 + 20 \times 22}$  的值。

11. If  $x$  and  $y$  are positive integers, and  $\frac{7}{2x} + \frac{3}{y} = 1$ , find the sum of possible values of  $x$ .

已知  $x$ 、 $y$  為正整數，求  $\frac{7}{2x} + \frac{3}{y} = 1$  中  $x$  的可能值之和。

12. If there exists distinct integers  $a$ ,  $b$  and  $c$  such that  $a+b+c=2015$  and  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2015}$ , find the value of  $(a-2015)(b-2015)(c-2015)$ .

若相異實數  $a$ 、 $b$ 、 $c$  使得  $a+b+c=2015$ 、 $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2015}$ ，求  $(a-2015)(b-2015)(c-2015)$  的值。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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Number Theory

數論

13. If  $x$  is a non-negative integer and  $3x + 4y \leq 250$ , find the maximum integral value of  $y$ .

若  $x$  為非負整數且  $3x + 4y \leq 250$ ，求  $y$  的最大整數值。

14. Simplify  $\frac{2356}{14645}$ .

化簡  $\frac{2356}{14645}$ 。

15. Find the integer closest to  $\sqrt{7 + \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}}$ .

求最接近  $\sqrt{7 + \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}}$  的整數。

16. Find the remainder when  $407^9 + 407^3$  is divided by  $407^2 + 408$ .

求  $407^9 + 407^3$  除以  $407^2 + 408$  的餘數。

17. If  $a$  and  $b$  are positive integers such that  $\sqrt{79 + 24\sqrt{7}} = a + b\sqrt{7}$ , find the value of  $a - 2b$ .

若  $a$ 、 $b$  為正整數使得  $\sqrt{79 + 24\sqrt{7}} = a + b\sqrt{7}$ ，求  $a - 2b$  的值。

18. Find the largest odd factor of each of  $50, 51, 52, \dots, 98, 99, 100$ , then find their sum. (Hint: Consider the scenario without 100)

求  $50, 51, 52, \dots, 98, 99, 100$  的每一個最大奇因數，由此求它們之和。(提示：先撇除 100 考慮)

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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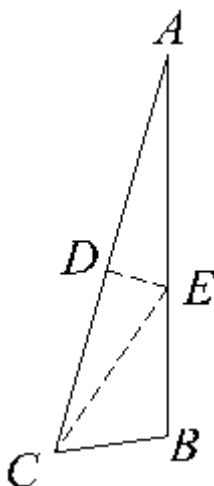
All answers should be written on the ANSWER SHEET.

Geometry

幾何

19. Refer to the triangle  $ABC$  below, David folds the triangle so that  $A$  overlaps  $C$  to form quadrilateral  $DEBC$  and folds the quadrilateral so that  $D$  overlaps  $B$ .  $DE$  is the trait after step 1 and  $CE$  is the trait after step 2. After that, the three smaller triangles completely overlaps, find  $\frac{AC^2}{AE^2}$ .

參考附圖，大偉將一三角形摺使  $A$  點與  $C$  點重疊來組成四邊形  $DEBC$  及  $D$  點與  $B$  點重疊。步驟一得摺痕  $DE$ ；步驟二得摺痕  $CE$ 。若由此法相摺後，三個三角形完全重疊，求  $\frac{AC^2}{AE^2}$  的值。



Question 19

第 19 題

20. Refer to the figure below,  $\triangle ACB$  is a right-angled triangle in which  $\angle ABC = 90^\circ$ .  $D, E$  are points on  $AC$  such that  $AD = DE = EC$ ,  $BD = 12$  and  $BE = 7$ . Find the value of  $5AC^2$ .

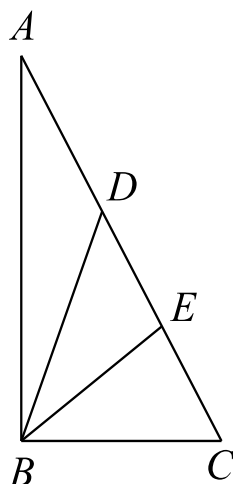
參考附圖， $\triangle ACB$  為直角三角形且  $\angle ABC = 90^\circ$ 。 $D$  和  $E$  是在  $AC$  上的點使得  $AD = DE = EC$ ，且  $BD = 12$  及  $BE = 7$ ，求  $5AC^2$  的值。

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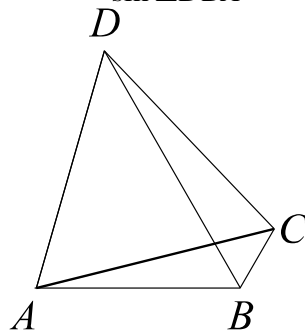


Question 20

第 20 題

21. Refer to the figure below,  $AB + BC = DB$  and  $CD = AD$ , find the value of  $\frac{\sin \angle DCA}{\sin \angle DBA}$ .

參考附圖， $AB + BC = DB$  且  $CD = AD$ ，求  $\frac{\sin \angle DCA}{\sin \angle DBA}$  的值。

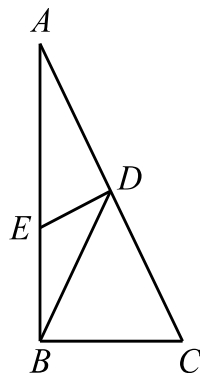


Question 21

第 21 題

22. Refer to the figure below,  $\angle AED = \angle ACB$ ,  $AD = DB = DC$  and  $AB:BC:AC = 12:5:13$ . Find  $AE:EB$ .

參考附圖， $\angle AED = \angle ACB$ 、 $AD = DB = DC$ 、 $AB:BC:AC = 12:5:13$ ，求  $AE:EB$ 。



Question 22

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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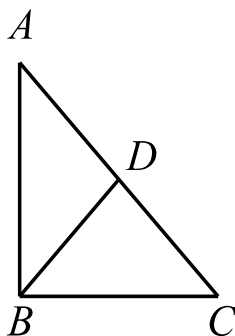
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第 22 題

23. Refer to the figure below,  $\triangle ADB \sim \triangle ABC \sim \triangle CDB$  and  $\angle DAB = x^\circ$ , find the value of  $x$ .

參考附圖，已知  $\triangle ADB \sim \triangle ABC \sim \triangle CDB$ ，且  $\angle DAB = x^\circ$ ，求  $x$  的值。

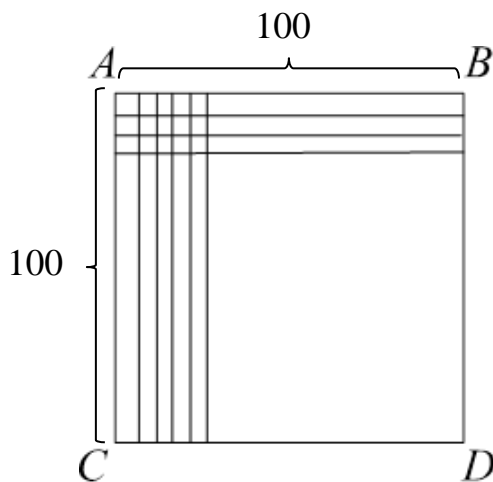


Question 23

第 23 題

24. Refer to the figure below, the figure is formed by 10000 identical squares. There are 100 squares in each column and each row. How many square(s) is / are there?

參考附圖，附圖的正方形由 10000 個全等的正方形組成，且每一直行、每一橫行均各有 100 個，問圖中有多少個正方形？



Question 24

第 24 題

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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請將答案寫在 

答題紙
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 上。

All answers should be written on the ANSWER SHEET.

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Combinatorics

組合數學

25. How many 4-digit number(s) whose sum of digits is 7 is / are there?  
有多少個四位數的數位之和是 7 ?
26. The lengths of the sides of a triangle are all integers and its perimeter is 12 units. Find the number of triangles to satisfy the condition.  
若三角形三條邊邊長均為整數，且周界為 12，求符合條件的三角形數目。
27. If non-negative integers  $x$  and  $y$  satisfy  $0.3x - 100 = 2015 - 4.7y$ , find the number of possible pairs of  $(x, y)$ .  
若非負整數  $x$ 、 $y$  符合  $0.3x - 100 = 2015 - 4.7y$ ，求  $(x, y)$  的解組數量。
28. Find the number of covering ways over  $2 \times 19$  grids with 19 small rectangles size  $2 \times 1$ .  
用 19 個  $2 \times 1$  的小長方形去覆蓋  $2 \times 19$  的方格網，共有多少種不同的覆蓋方法？


29. A palindromic number is a whole number that reads the same from either direction. For example, 1991

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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請將答案寫在 

答題紙
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 上。

All answers should be written on the ANSWER SHEET.

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and 23432 are two palindromic numbers. Find the value of the 21<sup>st</sup> 7-digit palindromic number in ascending order.

若某一整數的數位左右次序互換後數值不變，則稱該數為回文數，例如 1991 和 23432 均是回文數。  
求由小至大排列第 21 個 7 位回文數。

30. If  $0 \leq a \leq b \leq 20$  and  $a + b \geq 30$ , find the number of integral values of  $a - b$ .

若  $0 \leq a \leq b \leq 20$  且  $a + b \geq 30$ ，求  $a - b$  的整數可能值數量。

~ 全卷完 ~

~ End of Paper ~

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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