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世界國際數學競賽總決賽 2018

WORLD INTERNATIONAL MATHEMATICAL OLYMPIAD FINAL 2018

中學三年級 Secondary 3

時限：120 分鐘

Time allowed: 120 minutes

試題

Question Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 6 題，共 30 題，每題 5 分，總分 150 分，答錯不扣分。
   There are 5 exam areas and 6 questions in each exam area. There are a total of 30 questions in this Question-Answer Book. Each carries 5 marks. Total score is 150 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

本試題不可取走。
THIS Question-Answer Book CANNOT BE TAKEN AWAY.
未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。
DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

填空題（第 1 至 30 題）(每題 5 分，答錯及空題不扣分）

Open-Ended Questions (1^{st} ~30^{th}) (5 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. It is known that a prime number p can be written in the form of n^n+1 (n is an integer). If $p<10^{10}$, find the largest possible value of p.
   已知質數 p 可被寫成 n^n+1 的形式（n 為整數）且 $p<10^{10}$，求 p 的最大值。

2. It is known that $x=2+\cfrac{3}{2+\cfrac{3}{2+\cfrac{3}{2+\cdots}}}$. Find the value of x.
   已知 $x=2+\cfrac{3}{2+\cfrac{3}{2+\cfrac{3}{2+\cdots}}}$，求 x 的值。

3. It is known that $Z^3=8120601$. Find the integral value of Z.
   已知 $Z^3=8120601$，求整數 Z 的值。

4. If the sum of digits of a number is 17 and none of its digits is zero, we called it a 'good number'. Find the number of 5-digit 'good number(s)'.
   若一數的數位和為 17，且任何一個數的數位皆不是 0，我們則稱之為好數，求五位好數的數目。

5. Fruits are shared among students in class 9A, including apples, pears, oranges, strawberries, kiwis and watermelons. There are 5000 fruits of each type available. If each student picks two fruits randomly, at least how many students are there so that we are sure that two students picked the exactly two same fruits?
   9A 班同學分享水果，有蘋果、梨、橙、草莓、奇異果及西瓜 6 種，每種各 5000 個。如果每位同學任意拿兩個，那麼至少多少位同學拿過後才一定會出現兩人拿的水果是相同的。

6. If x、y are both real and $\begin{cases} y^2=x^3-2x^2+2x \\ x^2=y^3-2y^2+2y \end{cases}$, find the maximum value of $x+y$.
   已知 x、y 皆為實數，且 $\begin{cases} y^2=x^3-2x^2+2x \\ x^2=y^3-2y^2+2y \end{cases}$，求 $x+y$ 的最大值。

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Algebra
代數

7. It is known that α and β are roots of the equation $x^2-3x+7=0$. Find the value of $\alpha^2+2\beta^2+3\alpha+4$.
已知α和β分別為二次方程$x^2-3x+7=0$的兩個根。求$\alpha^2+2\beta^2+3\alpha+4$的值。

8. $f(x)$ is a polynomial function of degree 2. If $f(1)=0,\ f(3)=20,\ f(5)=64$, find the value of $f(7)$.
已知$f(x)$為一個二次多項式函數。若$f(1)=0,\ f(3)=20,\ f(5)=64$，求$f(7)$的值。

9. If x is real, find the smallest possible value of the expression $|x-7|+|x-4|+|x+5|+|x+6|+|x^2-7x+12|$.
若x為實數，求$|x-7|+|x-4|+|x+5|+|x+6|+|x^2-7x+12|$的最小值。

10. If the equation $2x^2-kx+k=0$ has more than 1 distinct real root, find the smallest positive integral value of k.
若方程$2x^2-kx+k=0$有多於一個相異實根，求k的最小正整數值。

11. If x is an integer and $\log x+\log(x+21)=2$, find the value of x.
若x為整數且$\log x+\log(x+21)=2$，求x的值。

12. Given that the cubic equation $3x^3+10x^2-81x-28=0$ has only two integral roots. Find the sum of those two integral roots.
已知一元三次方程$3x^3+10x^2-81x-28=0$只有兩個整數根。求該兩個整數根之和。

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Number Theory
數論

13. Find the remainder when 3^{2015} is divided by 365.
求 3^{2015} 除以 365 所得的餘數。

14. $2015^{\left(2014^{2013}\right)}{}_{(10)}$ is converted into a base-7 number . Find the unit digit of that base-7 number.
求十進制數 $2015^{\left(2014^{2013}\right)}{}_{(10)}$ 化為 7 進制後的個位數。

15. Find the sum of all integral roots of $(x+7)^5-(x-1)^5=2048$.
求 $(x+7)^5-(x-1)^5=2048$ 的所有整數解之和。

16. If n is a positive integer and $1+C_1^n+C_2^n+...+C_5^n=63$, find the value of n.
若 n 為正整數且 $1+C_1^n+C_2^n+...+C_5^n=63$ ，求 n 的值。

17. It is known that x is a positive prime number where $2x\equiv 3\,(\mathrm{mod}\,5)$. Find the smallest possible value of x.
已知 x 為正質數，且 $2x\equiv 3\,(\mathrm{mod}\,5)$ ，求 x 的最小值。

18. If x、y are positive primes which satisfy $x^2-9y^2=40$, find the value of $x-y$.
已知 x、y 皆為正質數，且 $x^2-9y^2=40$ ，求 $x-y$ 的值。

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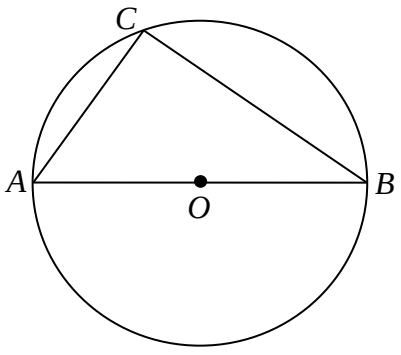
Geometry
幾何

19. If $0° \le \theta < 180°$ and $\dfrac{\sin\theta + 3\cos\theta}{\sin\theta - \cos\theta} = -1$, find the value of $\sin 2\theta + \tan 3\theta + \cos 4\theta$.
若 $0° \le \theta < 180°$ 及 $\dfrac{\sin\theta + 3\cos\theta}{\sin\theta - \cos\theta} = -1$，求 $\sin 2\theta + \tan 3\theta + \cos 4\theta$ 的值。

20. A rectangle with perimeter 24 is circumscribed in a circle whose radius is $2\sqrt{5}$ units. Find the area of the rectangle.
一個周界為 24 的長方形外接於一個半徑長度為 $2\sqrt{5}$ 的圓形，求該長方形的面積。

21. $A(1,6), B(6,7), C(7,2)$ are points on Cartesian coordinate system. Find the value of $\angle ACB$.
$A(1,6), B(6,7), C(7,2)$ 為平面直角坐標系上的三點，求 $\angle ACB$ 的值。

22. The figure below shows a circle with diameter AB. C is a point on the circle such that $OC^2 = AC \times BC$. Find the value of $\angle CAB$ if $AC < BC$.
參考附圖，AB 是圓的直徑。C 是圓上的一點使得 $OC^2 = AC \times BC$，若 $AC < BC$，求 $\angle CAB$ 的值。



Question 22
第 22 題

23. There is a circle whose radius is 1 cm. How many points are drawn on the circle to ensure that the distance between two points is less than or equal to $\sqrt{2}$ cm?
在一個半徑為 1 厘米的圓形上最少要劃上多少點，才可保證有最少有兩點的距離少於或等於 $\sqrt{2}$ 厘米？

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

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24. The diagonals of a trapezium are perpendicular to each other and the altitude of that trapezium is 5 units. If one of the diagonals is 13 units long, find the area of the trapezium.
已知一高為 5 的梯形的對角線互相垂直，若其中一條對角線的長度為 13，求該梯形的面積。

Combinatorics
組合數學

25. The members of a club have to join at least one of the three activities provided by the club, namely football, tennis and baseball. Each member has to join at least one of it, but cannot join all of them. Given the club has a total of 8 members and there are at least one participant for each activity, how many possible combination(s) are there for the members to join the activities?
在一個俱樂部裏，現有 3 個項目讓會員參加：足球、網球及棒球。俱樂部會員必須最少參與其中一項項目，但不能夠同時參加三項。如果俱樂部一共有 8 名成員，且每個項目都最少有一名參加者，一共有多少種分配方法？

26. Initially there are 5 red balls and 3 blue balls in a bag. Accidentally 1 ball is lost. Amy then draws 2 balls from the bag at random and both of the balls drawn are red. Find the probability that the lost ball is blue.
一個箱子裏有 5 個紅色球和 3 個藍色球，其中有 1 個球不見了。若小美再從這個箱子中抽出 2 個球，且 2 個球都是紅色，求最初不見了的球是藍色的概率。

27. Jack distributes 15 identical marbles into 5 distinct cans. If he knows that there are at least 6 marbles in one can, and there are at least one marble in each can, how many different way(s) is / are there for Jack to distribute the marbles？
小傑將 15 粒一樣的彈珠放到 5 個不同的罐子中。若已知其中一個罐子最少有 6 顆彈珠，且每個罐子最少有一顆彈珠，那麼一共有多少個分配彈珠的方法？

28. Find the number of covering ways over 2×20 grids with 20 small rectangles size 2×1.
用 20 個 2×1 的小長方形去覆蓋 2×20 的方格網，共有多少種不同的覆蓋方法？

29. A palindromic number is a whole number that reads the same from either direction. For example, 1991 and 23432 are two palindromic numbers. Find the value of the 22nd 7-digit palindromic number in ascending order.
若某一整數的數位左右次序互換後數值不變，則稱該數為回文數，例如 1991 和 23432 均是回文數。

求由小至大排列第 22 個 7 位回文數。

30. There are 8 balls each marked with one integer from 1 to 8 without repetition. The same applies to 8 baskets. Each ball is now randomly put into different baskets such that each basket has exactly 1 ball, find the probability that only one ball whose number differs from that on the basket.
將 8 張刻有不同數字 1 至 8 的數字球分別隨機投進 8 個刻有不同數字的籃子，求只有一個籃子裏的球的數字和籃子的數字不一樣的概率。

~ 全卷完 ~
~ End of Paper ~