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世界國際數學競賽總決賽 2018

WORLD INTERNATIONAL MATHEMATICAL OLYMPIAD FINAL 2018

高中組 Senior Secondary Group

時限：120 分鐘

Time allowed: 120 minutes

試題

Question Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 6 題，共 30 題，每題 5 分，總分 150 分，答錯不扣分。
   There are 5 exam areas and 6 questions in each exam area. There are a total of 30 questions in this Question-Answer Book. Each carries 5 marks. Total score is 150 marks. No points are deducted for incorrect answers.
3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.
Otherwise, contestant may be DISQUALIFIED.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。
All answers should be written on the ANSWER SHEET.

填空題（第 1 至 30 題）(每題 5 分，答錯及空題不扣分）
Open-Ended Questions (1st ~30th) (5 points for correct answer, no penalty point for wrong answer)

Logical Thinking
邏輯思維

1. Given that the sum of some mutually distinct positive integer is 90, find the maximum product of those numbers.
若干個互不相同的正整數的和為 90，求它們乘積的最大值。

2. After rearranging 8 digits of number 20150510, that number can be divided by 5^k, find the maximum number of k.
若把 20150510 八個數字重新整合成一個八位數字，使它能夠被 5^k 整除，求 k 的最大值。

3. It is known that $Z^3 = 8365427$. Find the integral value of Z.
已知 $Z^3 = 8365427$，求整數 Z 的值。

4. If $n \geq 3, a_n = 7a_{n-1} - 10a_{n-2} + 2^n$ and $a_1 = 10\frac{2}{3}, a_2 = 48\frac{2}{3}$, express a_n in terms of n.
若 $n \geq 3, a_n = 7a_{n-1} - 10a_{n-2} + 2^n$，且 $a_1 = 10\frac{2}{3}, a_2 = 48\frac{2}{3}$，以 n 表示 a_n。

5. It is known that $x = 1 + \cfrac{2}{3 + \cfrac{4}{1 + \cfrac{2}{3 + \ddots}}}$. Find the value of x.
已知 $x = 1 + \cfrac{2}{3 + \cfrac{4}{1 + \cfrac{2}{3 + \ddots}}}$，求 x 的值。

6. Given x is rational, p and q are primes and $px^2 - qx + p = 0$, find the value of $|pq|$.
已知 x 為有理數，p 和 q 皆為質數且 $px^2 - qx + p = 0$，求 $|pq|$ 的值。

Algebra
代數

7. If equation $(x-2)^2+(y+6)^2=(x+1)^2+(y-4)^2=(x+k)^2+(y-24)^2$ has no real solution, find the value of k.
若方程 $(x-2)^2+(y+6)^2=(x+1)^2+(y-4)^2=(x+k)^2+(y-24)^2$ 無實數解，求 k 的值。

8. Given a, b, c are positive integers and $\begin{cases} 6a-b-c=20 \\ a^2+b^2+c^2=1979 \end{cases}$, if a is a two-digit number, find the minimum value of $a+b+c$.
已知 a, b, c 皆為正整數且 $\begin{cases} 6a-b-c=20 \\ a^2+b^2+c^2=1979 \end{cases}$，若 a 為兩位數，求 $a+b+c$ 的最小值。

9. Find the value of $\sum_{k=1}^{10}\left(\frac{4k-1}{k(k+2)}\bullet 3^{k-1}\right)$.
求 $\sum_{k=1}^{10}\left(\frac{4k-1}{k(k+2)}\bullet 3^{k-1}\right)$ 的值。

10. It is known that the domain of $f(x)=\sqrt{10x-x^2-21}-\sqrt{2x-x^2+35}$ is defined as $a\le x\le b$. a and b are unknown constants. Find the minimum value of $f(x)$.
已知 $f(x)=\sqrt{10x-x^2-21}-\sqrt{2x-x^2+35}$ 的定義域為 $a\le x\le b$，a 和 b 為未知常數。已知 $a\le x\le b$，求 $f(x)$ 的最小值。

11. Given $f(x)=\frac{x}{1+3x}$, evaluate $f^{10}(2)$.
設 $f(x)=\frac{x}{1+3x}$，試求 $f^{10}(2)$ 的值。

12. Given x, y, z are real numbers and $\begin{cases} 4x-y-2z=8 \\ 3x+2y-z=12 \\ x+3y+2z=11 \end{cases}$, find the value of $x+y+z$.
已知 x, y, z 皆為實數，且 $\begin{cases} 4x-y-2z=8 \\ 3x+2y-z=12 \\ x+3y+2z=11 \end{cases}$。求 $x+y+z$ 的值。

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Number Theory
數論

13. Express $(\frac{1}{2}+\frac{\sqrt{3}}{2}i)^{2015}$ in the form of $a+bi$.
以 $a+bi$ 形式表示 $(\frac{1}{2}+\frac{\sqrt{3}}{2}i)^{2015}$ 。

14. Given x, y are positive integers and $4^x-3^y=1$, find the maximum value of $x-y$.
已知 x, y 皆為正整數，且 $4^x-3^y=1$。求 $x-y$ 的最大值。

15. Find the remainder of $(x^{15}-2x^2+4)\div(x^2-4)$. (Express your answer in descending order of x)
求 $(x^{15}-2x^2+4)\div(x^2-4)$ 的餘式。(以 x 的降冪表示答案)

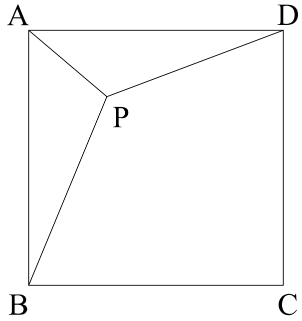
16. Given $\begin{cases} x^2+\{y\}^2=3.6 \\ y^2+\{x\}^2=3.2 \end{cases}$, find the maximum value of $x+y$.
已知 $\begin{cases} x^2+\{y\}^2=3.6 \\ y^2+\{x\}^2=3.2 \end{cases}$，求 $x+y$ 的最大值。

17. Given x is a positive integer less than 110 and $x^2-77x+146\equiv 0 \pmod{35}$, find the sum of all possible x.
已知 x 為小於 110 的正整數且 $x^2-77x+146\equiv 0 \pmod{35}$，求 x 的所有可能值之和。

18. Find the remainder when 2015^3 is divided by 2014×2016.
求 2015^3 除以 2014×2016 時的餘數。

Geometry
幾何

19. $ABCD$ is a square. Given $PA=1$, $PB=3$ and $PD=\sqrt{7}$, find the area of square $ABCD$.
$ABCD$ 是一個正方形，$PA=1$ 、$PB=3$、$PD=\sqrt{7}$，求正方形 $ABCD$ 的面積。

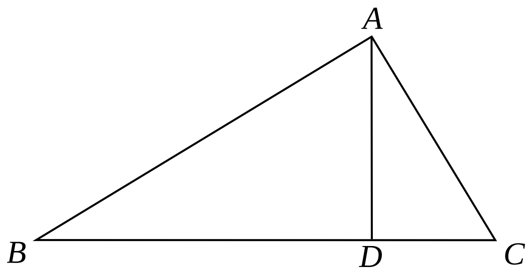


Question 19
第 19 題

20. It is known that the coordinates of six vertices of a convex hexagon are $(0,0)$, $(0, 6)$, $(2, -1)$, $(2, 8)$, $(3, 4)$, $(3, 7)$, find the area of that hexagon.
已知一個凸六邊形的六個頂點在直角坐標上的坐標分別為
$(0,0)$, $(0, 6)$, $(2, -1)$, $(2, 8)$, $(3, 4)$, $(3, 7)$。求該六邊形的面積。

21. The equation of a circle C is $C: x^2+y^2+2x-4y-15=0$. A straight line passing through $A(6,1)$ intersects the circle at two points X and Y. If the distance between A and X and the distance between A and Y are respectively p and q, find the value of pq.
對於圓 $C: x^2+y^2+2x-4y-15=0$，一直線經過 $A(6,1)$ 並與圓 C 相交於兩點 X 和 Y。若 A 點與 X 點及 Y 點的距離分別為 p 及 q，求 pq 的值。

22. In a right-angled triangle ΔABC, $\angle BAC=90°$ and AD is the altitude of the triangle to the hypotenuse. If the length of three sides of the triangle are all integers and $CD=1331$, find the perimeter of ΔABC.
在直角三角形 ΔABC 中，$\angle BAC=90°$ 及 AD 是斜邊上的高。已知 ΔABC 的三邊長度皆為整數，且 $CD=1331$，求 ΔABC 的周界。



Question 22
第 22 題

23. Draw and a regular hexagon and a regular octagon inscribed in one circle. If the area of octagon is 8 cm^2, find the area of the hexagon. (Express your answer in surd form.)
在一個圓形內繪畫一個圓內接正六邊形和一個圓內接正八邊形，若正八邊形的面積為 8 平方厘米，正六邊形的面積是多少平方厘米？（以根式表示答案）

24. Given $\sin\theta-\cos\theta=\dfrac{1}{3}$, find the value of $\sin^3\theta-\cos^3\theta$.
已知 $\sin\theta-\cos\theta=\dfrac{1}{3}$，求 $\sin^3\theta-\cos^3\theta$ 的值。

<u>Combinatorics</u>
<u>組合數學</u>

25. By using positive integer P to simplify $\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+\cdots+\dfrac{P-1}{P!}$.
簡化並以正整數 P 表示 $\dfrac{1}{2!}+\dfrac{2}{3!}+\dfrac{3}{4!}+\cdots+\dfrac{P-1}{P!}$。

26. Nick has 3 bags. There are 3 red balls and 5 yellow balls inside each bag. If randomly choosing 1 ball from each bag, find the probability of giving exact 2 yellow balls.
<u>小力</u>有 3 個袋，每個袋內均有 3 個紅球和 5 個黃球。若從每個袋子隨機抽一個球出來，求剛好抽出 2 個黃球的概率。

27. Put 7 different envelopes in 4 mail boxes. Given that each mail box contains at least one envelop, how many way(s) is / are there?
將 7 封不同的信件隨機投入 4 個郵筒，而每個郵筒都至少投入一封，有多少種不同的投法？

28. The sum of 2 positive integers is 147. Now the product of these 2 numbers is an integral multiple of the sum of these 2 numbers. Find the number of possible solution(s).

兩個正整數之和為 147，且該兩數之積為兩數之和的整數倍數。求該兩數的所有解數量。

29. A palindromic number is a whole number that reads the same from either direction. For example, 1991 and 23432 are two palindromic numbers. Find the value of the 23rd 7-digit palindromic number in ascending order.

若某一整數的數位左右次序互換後數值不變，則稱該數為回文數，例如 1991 和 23432 均是回文數。求由小至大排列第 23 個 7 位回文數。

30. Given a, b are both positive integers less than 1000 and $5a^2 = b^2 + 2$, how many pair(s) of (a, b) is / are there?

已知 a, b 皆為少於 1000 的正整數且 $5a^2 = b^2 + 2$，求符合算式的數組 (a, b) 的數目。

~ 全卷完 ~

~ End of Paper ~