1. The remainder when 2010^{2011} is divided by 30 is
   (A) 0 (B) 1 (C) 5 (D) 10 (E) 11

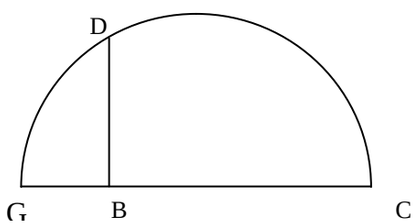
2. Tharindu brought a cake on his birthday. Supun had two fifths of the cake and Nipuna had half of the remaining part. What fraction of the cake did Nipuna have?
   (A) 1/10 (B)1/5 (C) 3/5 (D) 3/10 (E) 2/5

3. Which of the following numbers is the smallest?
   (A) $\frac{1}{\sqrt{2011}+\sqrt{2010}}$ (B) $\frac{1}{\sqrt{2010}+\sqrt{2009}}$ (C) $\frac{1}{\sqrt{2009}+\sqrt{2008}}$ (D) $\frac{1}{\sqrt{2008}+\sqrt{2007}}$
   (E) $\frac{1}{\sqrt{2007}+\sqrt{2006}}$

4. The product of two positive integers is 24. What is the least possible value of the sum of the two integers?
   (A) 14 (B)12 (C) 11 (D) 10 (E) 9

5. Which of the following numbers is the largest?
   (A) $\sqrt{\sqrt[3]{7\times 11}}$ (B) $\sqrt{11\times\sqrt[3]{7}}$ (C) $\sqrt{7\times\sqrt[3]{11}}$ (D) $\sqrt[3]{7\times\sqrt{11}}$
   (E) $\sqrt[3]{11\times\sqrt{7}}$

6. In the semicircle given below $AB = 4$ and $BC = 9$ and BD is perpendicular to AC. What is the length of BD?



   (A) 6.5 (B) 6 (C) 5 (D) 8 (E) None

7. Let N be the largest positive integer n such that $\frac{n}{n+1} < \frac{2004}{2011}$. What is the product of the digits of N?
   (A) 36 (B) 80 (C) 96 (D) 112 (E) 128

8. At the New Year ceremonial first meal at Shanthini's home each person ate one seventh of a plate of *Laddu*s and one fifth of a plate of *Kevuns*. If altogether 24 plates of sweets, *Laddus* or *Kevuns*, were consumed, how many people were there?
   (A) 140 (B) 70 (C) 48 (D) 35 (E) 24

29. Let f be the relation defined by $f(n) =$ The smallest interior angle value of the n sided polygon with perimeter n units with maximum area, for each positive integer $n(>2)$. Now consider the following statements.

    I. Given two positive integers $n, m\ (>2)$ with$n > m$ we always have that $f(n) > f(m)$.
    II. Given a positive integers n we always have that the difference between $f(n)$ and $f(n+1)$ is less than $45°$.
    III. There exist two distinct positive integers $n, m\ (>2)$ such that the difference between $f(n)$ & $f(m)$ is less than $1°$.

    Which of the above is/are true?

    (A) I only (B) II only (C) I and II only (D) II and III only (E) All are correct

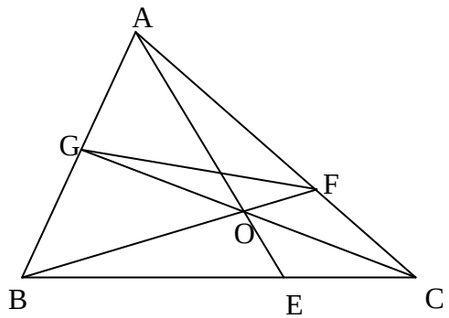
30. Let $x_n = \sqrt{6+\sqrt{6+\sqrt{6+\cdots+\sqrt{6}}}}$ (here 6 appears n times) for each positive integer n. Consider the following expressions.

    I. $x_n < 3$ for infinitely many positive integers n.
    II. $x_{2011} > 2.5$.
    III. There are only finitely many n such that $x_n > 2.999\ldots9$ (Here 9 appears 2011 times).

    Which of the above is/are true?

    (A) I only (B) II only (C) I and II only (D) II and III only (E) All are correct

25. In the given figure $\frac{AG}{GB} = \frac{AF}{FC} = 2$. It is also given that $AB = 16,\ BC = 30$ and $AC = 24$. What is the length of BE?



(A) 10 (B) 12 (C) 15

(D) 16 (E) 20

26. Let $S_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!}$ where n is an integer and $n! = 1 \times 2 \times \ldots \times (n-1) \times n$. What is the value of $\frac{1 - S_{2011}}{1 - S_{2012}}$?

(A) 2010 (B)2011 (C) 2012 (D) 2013 (E) 2013!

27. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers such that, $a_n = 1$ if n is a perfect square and $a_n = k$ if $(k-1)^2 < n < k^2$ where k is a positive integer. Which of the below is/are true?

I. For each positive integer k we can find a positive integer $n(> k)$ having the property $a_n = 1$.
II. Number of positive integers n such that $a_n = k$ is greater than or equal to k.
III. There are at least 2011^{2011} positive integers n such that $a_n = a_{2011^{326}}$.

(A) III only (B) I and II only (C) I and III only (D) II and III only (E) All are correct

28. The number of pairs (m, n) of integers that satisfy $m^3 + 6m^2 + 5m = 27n^3 + 9n^2 + 9n + 2011$ is,

(A) Infinitely many (B) 2011 (C) 27 (D) 9 (E) None

9. The numbers $1, 2, 3, \ldots, 2010, 2011$ are written on a white board. Now carry out the following operation: Erase two numbers x and y and replace them by $x + y - \frac{1006}{2010}$. What number would remain on the board if this operation is carried out till only one number remains on the board?

(A) 2011 (B) 1005×2010 (C) 1005×2011 (D) 1006×2010 (E) 1006×2011

10. The fraction $\frac{2022}{2011}$ can be written in the form $1 + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}}$ where (a, b, c, d) equals

(A) (182, 1, 4, 1) (B) (182, 1, 4, 2) (C) $(182, 1, 2, 1)$ (D) (181, 1, 4, 1)
(E) (183, 1, 4, 1)

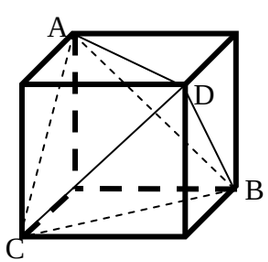
11. In a motor car race, Lajanugen finishes the 10 kilometer race first. When Lajanugen crosses the finish line, Gamlath and Schumacher are 400 m and 2800 m respectively behind the finish line. If Gamlath and Schumacher maintain their previous average speeds for the remaining part of the race, by how many meters will Gamlath cross the finish line ahead of Schumacher?

(A) 2400 (B) 2450 (C)2500 (D) 2550 (E) 2600

12. A triangle has integral side lengths and its perimeter is 7. What is the maximum possible area of the triangle?

(A) $\frac{\sqrt{7}}{2}$ (B) $\frac{\sqrt{7}}{4}$ (C) $\frac{3\sqrt{7}}{4}$ (D) $\frac{\sqrt{35}}{4}$ (E) $\frac{\sqrt{35}}{2}$

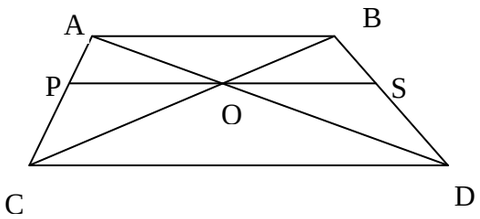
13. What is the ratio of the surface area of the cube to that of the inscribed tetrahedron ABCD?



(A) $\frac{3+\sqrt{3}}{6}$ (B) $2\sqrt{3}$ (C) $\sqrt{3}$

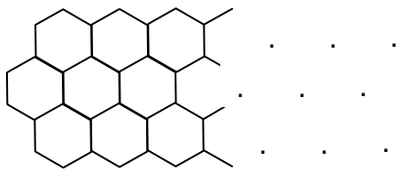
(D) $\frac{3}{2\sqrt{3}}$ (E) 3

14. In the trapezoid below AB is parallel to CD and O is the intersection of AD and BC. The line PS is drawn through O in such a way that PS is parallel to DC. If $AB = 20$ and $CD = 30$, what is the length of PS?



(A) 21 (B) 24 (C) 25 (D) 27 (E) 28

15. A pattern of hexagons is made out of matches as follows. Each straight line segment is a match.



What is the maximum number of hexagons that can be made out of 2011 matches?
(A) 540 (B) 543 (C) 546 (D) 547 (E) 548

16. In the *Land of Liars*, *White* clansmen always tell the truth, *Red* clansmen always lie. Nimal, Bimal, Wimal, Susil and Sunil, who belong to either the *White* or *Red* clan, not both, in conversation say the following.

Nimal : "Susil is a *White*."
Bimal : "Sunil is a *Red*."
Wimal : "Nimal is not a *Red*."
Susil : "Bimal is not a *White*."
Sunil : "Nimal and Wimal are in different clans."
How many are *Reds*?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

17. Sameera has 4 coins; one 1 Cent coin, one 5 Cents coin, one 10 Cents coin, and one 25 Cents coin. What is the total number of different non zero amounts that can be made from some or all of these coins?

(A) 16 (B) 15 (C) 10 (D) 8 (E) 6

18. Which of the following is a prime?

(A) $7^{241} - 5^{191}$ (B) $7^{241} + 5^{191}$ (C) $2^{2^4} + 1$ (D) $2^{2^4} - 1$
(E) $7^{242} - 5^{191}$

19. Yadavan has one Rupee, 50 Cents and 25 Cents coins in his wallet; altogether 20 coins. If the amount in his purse is Rs. 12.50 and there are more 25 Cents coins than 50 cents coins, how many 25 Cents coins does he have?
(A) 2 (B)4 (C) 6 (D) 8 (E) 10

20. How many integers n are there such that the difference between $\sqrt{n}$ and $\sqrt{2011}$ is less than 1?

(A)176 (B)177 (C)178 (D)179 (E)180

21. How many integers n are there such that $(n^2 + 1)(n + 2)^2$ is a perfect square of an integer?

(A) None (B) 1 (C) 2 (D) 3 (E) 4

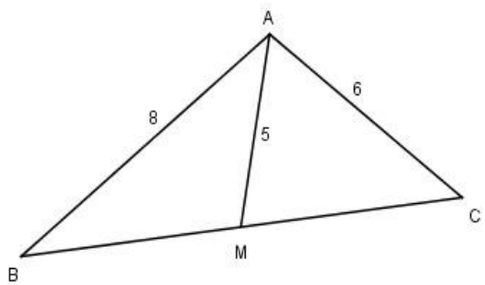
22. How many integers n are there between 1 and 2011 inclusive of 1and 2011 such that $1^n + 2^n + 3^n + 4^n + 5^n$ is divisible by 5?

(A) 1505 (B) 1506 (C) 1507 (D) 1508 (E) 1509

23. Consider the sequence defined $1, 2, 2, 3, 3, 3, \ldots . n, n, n, \ldots, n, \ldots$ (here n appears n times) Find the 2011th term of the sequence.

(A) 60 (B) 61 (C) 62 (D) 63 (E)64

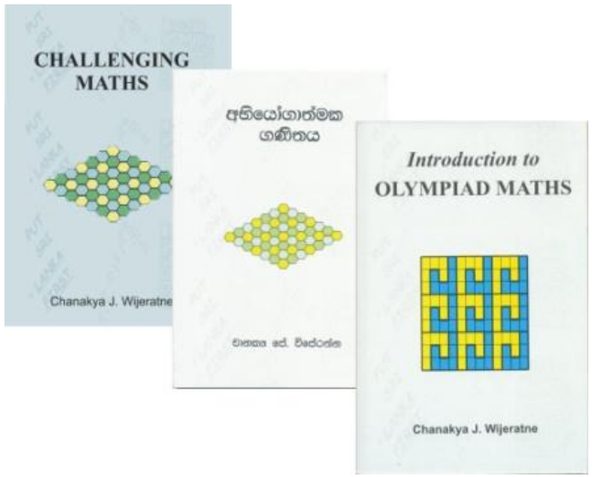
24. *AM* is a median of the triangle in the figure. What is the area of the triangle?



(A) 20 (B) 24 (C) 25 (D) 30 (E) 32

Thank you very much for your participation in the *SLMSC 2011*. Your score on this competition will be posted against your index number in www.slmathsolympiad.org. The best 40 students in the *SLMSC 2011* will be invited (they will be notified by mail) to participate in the *SLMCSC 2011* which will be held on 9th April 2011. In this competition we have tried to showcase mathematics by posing puzzle type questions covering various areas of mathematics. Though the problems require very little knowledge, not more than a Year 10 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your mathematical interest beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! Look for opportunities - there are many websites in the internet and also good books featuring excellent mathematical problems - challenge yourself! For any comments/ suggestions:

e-mail:cjw@maths.cmb.ac.lk, sms or call: 072 3678215



Index No:

Medium **ENGLISH**

SRI LANKAN MATHEMATICS SENIOR COMPETITION 2011

March 26, 2011

This question paper has **30 questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

Scoring System for the Sri Lankan Mathematics Competition

Questions 1 to 10 : 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20 : 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30 : 8 points for correct response, 3 points for no response, and 0 points for incorrect response.

Sri Lanka Olympiad Mathematics Foundation