1. a and $\left(a+\frac{1}{a}\right)$ are both positive integers. How many values can a take?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. Two identical rectangles are divided into three and two equal parts as shown. Let the shaded areas of figures 1 and 2 be A and B respectively. How many B's are there in A?

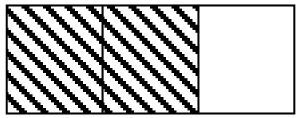


Figure 1

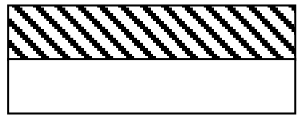


Figure 2

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) $1\frac{1}{3}$ (E) $1\frac{2}{3}$

3. In a decimal number, a bar over one or more consecutive digits means that the pattern of digits under the bar repeats without an end. For example, $0.\overline{387} = 0.387387387...$

Which of the following is the largest decimal number?

(A) $0.\overline{71}$ (B) $0.\overline{717}$ (C) $0.\overline{7177}$ (D) $0.\overline{71771}$ (E) $0.\overline{7171}$

4. Sarath and Kamala each have a collection of marbles. If Sarath gives 4 marbles to Kamala, then they would both have an equal number of marbles. If Kamala gives 4 marbles to Sarath then Sarath would have 3 times as many marbles as Kamala.

How many marbles does Sarath have?

(A) 12 (B) 16 (C) 20 (D) 24 (E) 28

5. Dileepa takes a wooden cube with side length 2013 cm and colours it red. Then, he cuts it into 2013^3 smaller cubes with side length 1 cm. How many cubes are there with exactly two faces coloured red?

(A) 2011×8 (B) 2013×8 (C) 2011×12 (D) 2013×12 (E) (2013×2013 − 4)×6

6. In the initial round of a soft ball cricket tournament, 45 matches are played where every participating team plays a match against each of the other teams exactly once. How many teams are there in the tournament?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

26. For real numbers a and b which are not both zero, define $a \oplus b$ by $a \oplus b = \frac{a^2b+b^2a}{a^2+b^2}$. Which of the following is/are true?
I. For real numbers a and b, if $a \oplus b = a$ then $a = b$.
II. There exist a, b natural numbers such that $a \oplus b = 2013$.
III. There are only a finite number of 2-tuples (a, b) such that $a \oplus b \geq a + b$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) All

27. Consider the infinite sequence 6, 96, 996, 9996, … , 9999…96,… . In the n^{th} term of the sequence the digit 9 appears $(n-1)$ times. Which of the following statements are true?
I. There exists a perfect square in the sequence.
II. There exist infinitely many terms in the sequence which are divisible by 8.
III. If a term in the sequence is divisible by 8 then it is also divisible by 16.

(A) I only (B) III only (C) I and II only (D) II and III only (E) None
.

28. Let $f(n) =$ Number of even positive divisors of n – Number of odd positive divisors of n ; where n is a natural number. Which of the following statements regarding f is/are true?
I. $f(10) = 0$
II. There exists a natural number n such that $f(n) =2013$.
III. $f(n) < 0$ for half or more of the natural numbers from 1 to 2012^{2013}.

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) All

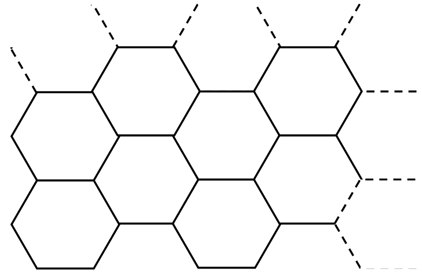
29. A positive integer is said to be *cool* if it can be written as a sum of two or more consecutive positive integers. Which of the following statements is/are true?
I. 2013 is a *cool* number.
II. If a number is not *cool* then all of its multiples are not *cool.*
III. There are infinitely many numbers which are not *cool.*

(A) I only (B) II only (C) I and III only (D) II and III only (E) All

30. Two persons A and B are each wearing a hat with a number written on it. Neither A nor B can see the number on their own hat but they can see the number on the other's. A sees number 5 on B's hat and B sees number 4 on A's hat. They are told that A has the product of two positive integers written on her hat and that B has the sum of the same two numbers written on her hat.
First B is asked whether she knows for sure what the two numbers are. If her answer is 'no', A is asked the same question and so on until someone answers 'yes'. Assuming that both are perfect logicians and answer truthfully, who says 'yes' first and when?

(A) B on her 2^{nd} question
(B) B on her 3^{rd} question
(C) A on her 1^{st} question
(D) A on her 2^{nd} question
(E) No one will answer 'yes'

21. In the given tessellation of identical regular hexagons, the side-length of a hexagon is equal to 1 cm. Which of the following cannot be the distance between two centers of hexagons?



(A) 3 cm (B) $3\sqrt{3}$ cm (C) $2\sqrt{3}$ cm
(D) $\sqrt{21}$ cm (E) $\sqrt{6}$ cm

22. A 4-tuple (p, q, r, s) is said to be *good* if p, q, r and s are distinct positive integers satisfying the equation $p^3 + q^3 = r^3 + s^3$. For example (9, 10, 1, 12) is *good*. How many good 4-tuples are there?

(A) 1 (B) 4 (C) 8 (D) 32 (E) Infinitely many

23. In the Land of Liars, there are exactly 3 clans. Black clansmen always tell lies. White clansmen always speak the truth. Red clansmen are sometimes truthful. A, B and C are three people from the clans Black, White and Red not necessarily in the given order. D is a clansman from the Land of Liars. They were asked what their clan is and then what clan D is from. Their responses were,

A. I'm not a White; D is a Black.
B. I'm not a Black; D is a Red.
C. I'm not a Red; D is a White.
D. I'm not a Black; I am a White.

Which of the following can be concluded?
(A) D is a Red.
(B) D is a White and C is a Red.
(C) A is a Red, B is a Black, C is a White and D is a White.
(D) C is a White and D is a Red.
(E) B is a Red.

24. Given below is a correctly worked out addition problem where the letters A, D, E, I, L, Q, R, S and U represent different digits from 0 to 9. E and S are non-zero digits.

	E	Q	U	A	L	
	S	I	D	E	S	+
S	Q	U	A	R	E	

What is the maximum value that the number "SQUARE" can take?

(A) 105348 (B) 106439 (C) 107329 (D) 107458 (E) 108239

25. For how many distinct positive integral values of p can positive integers q, r and s be found such that $p^q + p^r = p^s$?

(A) Zero (B) One (C) Two (D) Three (E) Infinitely many

7. How many two digit numbers are 4 times the sum of its digits?

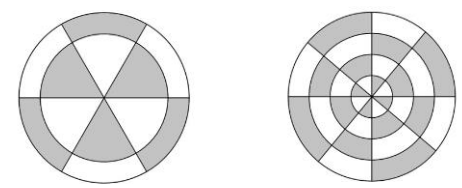
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. A sand timer consists of two connected glass balls. The passage of time is measured by allowing sand to trickle down from top to bottom. Given two sand timers of 4 and 9 minutes, which of the following is/are true?

I. An exact time of 13 minutes can be measured.
II. An exact time of 16 minutes can be measured.
III. An exact time of 29 minutes can be measured.

(A) I only (B) II only (C) III only (D)I and II only (E) All

9. Dartboard 1 contains concentric circles with radii 3 and 4 units. Dartboard 2 contains concentric circles with radii 1, 2, 3 and 4 units. The largest circles in each dartboard are divided into 6 and 8 equal sectors respectively and then shaded as below. *Anwar*, *Balachandran* and *Charith* are playing a game of chance. They shoot darts at Dartboard 1 or Dartboard 2. When they hit a shaded area they gain 1 point. Each player has an equal probability of hitting any spot on each of the dartboards. *Anwar* shoots darts only at dartboard 1, *Balachandran* only at dartboard 2 and *Charith* at both the dartboards 1 and 2, an equal number of times at each. Each player shoots 10 darts. Let $P(A)$ be the probability of *Anwar* winning, $P(B)$ the probability of *Balachandran* winning and $P(C)$ the probability of *Charith* winning. Which one of the following statements is true?



Dartboard 1 Dartboard 2

(A) $P(A) = P(C) = P(B)$
(B) $P(B) < P(C) < P(A)$
(C) $P(A) < P(C) < P(B)$
(D) $P(B) < P(C) = P(A)$
(E) $P(C) < P(A) = P(B)$

10. Your SLMC 2013 booklet has 8 pages. How many ways are there to reach the end of the booklet starting from the first page and without flipping leaves backwards and without going through a page twice?

(A) 4 (B) 7 (C) 8 (D) 15 (E) 16

11. Three friends want to buy exercise books given at hugely discounted prices of Rs. 45, Rs. 50, Rs. 65, Rs. 70 and Rs. 100 from the school bookstore. The bookstore owner however prefers having more customers than having a few customers buying several exercise books. So, a customer buying his x^{th} exercise book has to pay x times the original price. The three friends who have not bought any exercise books from the bookstore before, decide to buy a total of 5 books, one from each type. What is the minimum cost at which they could buy all the 5 different exercise books?

(A) 270 (B) 305 (C) 330 (D) 365 (E) 425

12. There are 4 balls named A, B, C and D arranged in the given order in a row. Two operations are performed on this row of balls.

Operation 1: Take the first ball and place it before the last one.
Operation 2: Take the last ball and place it after the first one.

These operations are performed alternatively, with the first step being operation 1. What would be the arrangement of the balls after step 2013?

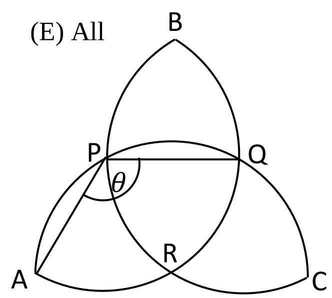
(A) ABCD (B) BCAD (C) BDCA (D) DCBA (E) None of the given

13. Which of the following is/are correct?

I. 1/3 in base 3 is 0.1
II. 1/3 in base 10 is $0.\overline{3}$
III. 1 equals $0.\overline{9}$ in base 10.

(A) I only (B) II only (C) III only (D) I and II only (E) All

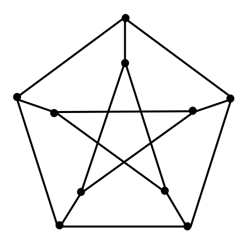
14. $APQC, ARQB$ and $BPRC$ are identical semi circles lying on a plane.
Let $\measuredangle APQ = \theta$. (As shown in the figure which is not drawn to scale)
What is the value of θ?



(A) 120° (B) 135° (C) $50\sqrt{6}°$
(D) 105° (E) 108°

15. In the following diagram, the 10 indicated nodes (dots) have to be coloured so that no two nodes of the same colour share an edge (a straight line joining two nodes). What is the minimum number of colours required to colour the nodes?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5



16. Which of the following fractions can be written as a terminating decimal?

(A) $\frac{2013}{2^{2013}}$ (B) $\frac{2^{2013}}{2013\times 5^{2013}}$ (C) $\frac{5^{2013}}{2013\times 2^{2013}}$ (D) $\frac{2^{2013}\times 5^{2013}}{2013}$ (E) $\frac{2013}{3^{2013}}$

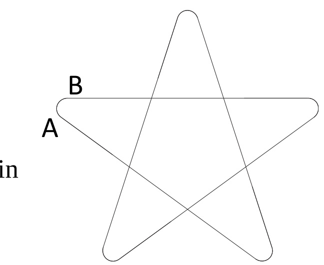
17. $\{3.7, 4.1, a, 8.5, 9.2, 2a\}$
The six numbers shown are listed in increasing order. Which one of the following lists does not contain an impossible value for the range (= maximum value – minimum value) of the six numbers given above?

(A) 4.0, 5.2, 7.3, 11.6, 12.9
(B) 5.1, 7.5, 11.1, 12.3, 14.0
(C) 7.3, 11.6, 12.2, 14.1, 15.3
(D) 5.8, 8.1, 11.6, 12.9, 13.3
(E) 5.4, 7.3, 10.6, 12.9, 13.0

18. In a grocery store, oranges are stacked in a pyramidal way such that the oranges in the $(k + 1)^{th}$ layer are stacked by filling all the pockets (formed by 4 oranges) in the k^{th} layer. The oranges are stacked with a base of $m \times n$ oranges (where $m \geq n$) in the shape of a rectangle with n oranges as its width and m oranges as its length, in layers one above the other until no more oranges can be stacked. How many oranges are in the top layer?

(A) $m - n - 1$
(B) $m - n$
(C) $m - n + 1$
(D) $m + n$
(E) $m + n + 1$

19. Manuja used a wire to make a frame of a regular pentagram (A regular pentagram is a star formed by extending the sides of a regular pentagon). He kept the wire straight for the sides and at the corners he bent the wire in circular arcs. AB in the figure is one such arc.



What is the angle subtended by the arc AB on its center?

(A) 36° (B) 72° (C) 108° (D) 144° (E) 162°

20. Two players play a game with two piles of 5 stones and 18 stones. At each turn, the player takes from the bigger pile a non-zero multiple of the number of stones in the smaller pile. The first player to empty a pile is the winner. They take turns alternatively. Which of the following is true?

(A) The first player has a winning strategy.
(B) The second player has a winning strategy.
(C) The game will not end.
(D) No player has a winning strategy.
(E) The second player always loses.